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INVESTIGATION OF FACTORS WHICH CONTRIBUTE TO MALLAUNCH OF FREE ROCKETS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The simulation of the motion of a single-round launcher/rocket system during launch, transition to free flight, and free flight of the rocket is considered. A simple physical model for the system is described. The model consists of an arbitrary rigid-body launcher, with 3° of rotational freedom, and an arbitrary rigid-body rocket, which is attached to the launcher during the spin-up (if applicable), detent, and guidance phases by "springs." These springs are intended to model the flexibility and damping characteristics of

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Abstract (Concluded)

the rocket's shoes or the interface between the rocket and the tube of a tube-type launcher. The model also includes the effects of thrust malalignment, tipoff, variable thrust, and spin torque programs and friction.

Using modern vector/matrix algebra, the vector equations are converted to matrix equations, amenable to digital programming and solution. Results of preliminary simulation runs made using a computer code written to perform the required numerical integration are presented and discussed.

CONTENTS

Section I. INTRODUCTION

The mallaunch of free rockets continues as a significant problem. Each new system, i.e., each new launcher/rocket combination, possesses unique characteristics which must be analyzed and understood before the performance of the system can be predicted or optimized. The analyses which are conducted are usually both analytical and experimental. Simple physical models of the actual system are usually devised and mathematical theories of how the system will behave are constructed on the bases of such physical models. The behavior of a launcher/rocket system predicted by such a theory is normally compared with experimental data. By doing this, improvements in both the theory and the design of the system can usually be made. Furthermore, once a theory which predicts the behavior of a system or class of systems is devised, analytical experiments using the theory often give impetus to the development of radically new systems.

The work reported herein represents a first step in developing a theory for the dynamical behavior of free rocket launch systems. As such, the results of this study should be useful in the ways previously mentioned. In particular, this study appears to be the first in which the motion of rocket/launcher systems during the detent, guidance, tip-off, and free-flight phases of the complete flight is considered.

Studies of various aspects of free rocket launch dynamics have already been made [1,2,3]. Because of the advances in computational methods and simulation of dynamic systems in general it is important that such a study be conducted. This report is a contribution in this regard.

In Section II, a simple physical model for a single-round launcher/rocket system is described. This model includes the effects of many factors which contribute to mallaunch of free rockets and possesses enough flexibility to allow for addition of the effects of other factors.

Differential equations, the solution of which provides a description of the motion of the system, are derived in Section III using the Eulerian vector approach. The vector equations which are obtained are converted to matrix equations amenable to digital solution by using modern matrix/vector algebra. As a result of such conversion, the burden of performing much of the algebra involved in computing derivatives of the dependent variables may be placed on the digital computer.

Section IV provides descriptions of some of the methods used in implementing the digital program. In particular, the manner in which spin torque and thrust profiles are generated, matrix algebra is performed and the integration of the equations is carried out are explained.

Typical results obtained using the digital computer program (Fortran IV computer code) are presented and discussed in Section V and a summary and conclusions based on the study are presented in Section VI. The appendices are descriptions of the input needed for the program and a listing of the computer code.

Section II. PHYSICAL MODEL FOR THE SYSTEM

a. General Comments

The physical model adopted for the current study is described in this section. The model is fairly general in that it allows for rotational motions of an arbitrary, rigid-body launcher and arbitrary (six degrees of freedom when not constrained by the launcher and five degrees of freedom when constrained) motion of the rocket which is modeled as a constant mass rigid body. The model includes the effects of:

1) Thrust malalignment.
2) Large initial launch angle (elevation).
3) Mass unbalance (both static and dynamic) of the rocket.
4) Flexibility of the launcher/rocket interface.
5) The detent force.
6) Gravity tipoff.
7) Variable spin and thrust programs.
8) Friction.

Figure 1 is a graphical depiction of the complete model. In that figure is shown a rocket of mass m, which rests upon a launcher of mass M. The rocket is connected to the launcher by mechanical devices which rigidly constrain the rocket's relative motion along the prescribed line of flight (through a detent device which is assumed to act at the rear rocket support) and flexibly constrain relative pitch and yaw motions of the rocket. The system of two rigid bodies adopted for the model has up to nine degrees of freedom, with eight during the detent phase.

In the following sections, the launcher and rocket models are treated separately and relevant definitions are given.

b. Launcher Model

The launcher model may be represented by the single rigid body shown in Figure 2. This body has three degrees of rotational freedom about the fixed point 0 which, in general, does not coincide with the launcher's center of mass C_L. The coordinate system* OXYZ, which is inertial and has its Z-axis directed vertically down and its X-axis

*All coordinate systems used or referred to herein are dextral, orthogonal systems.

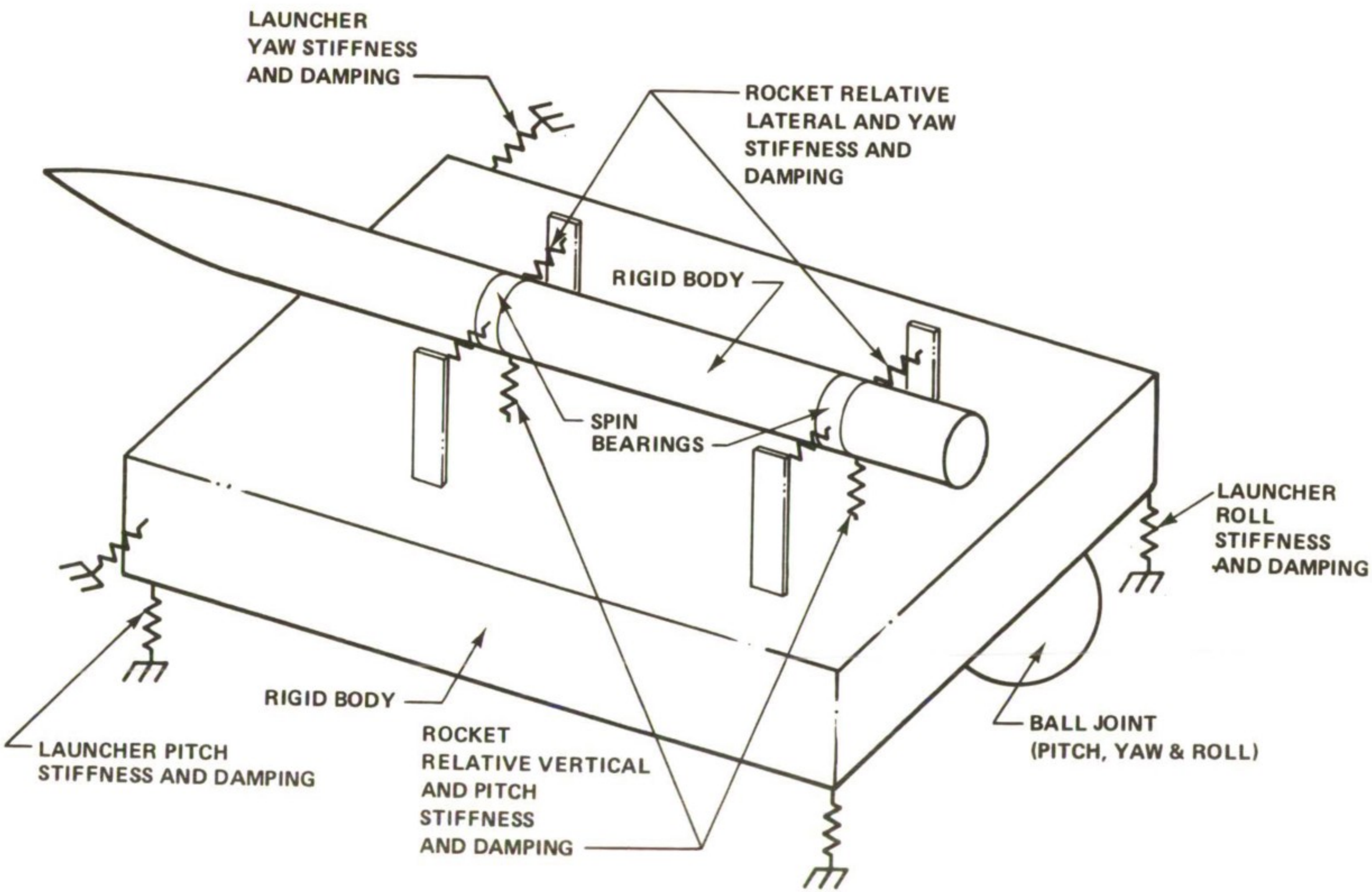

Figure 1. System model.

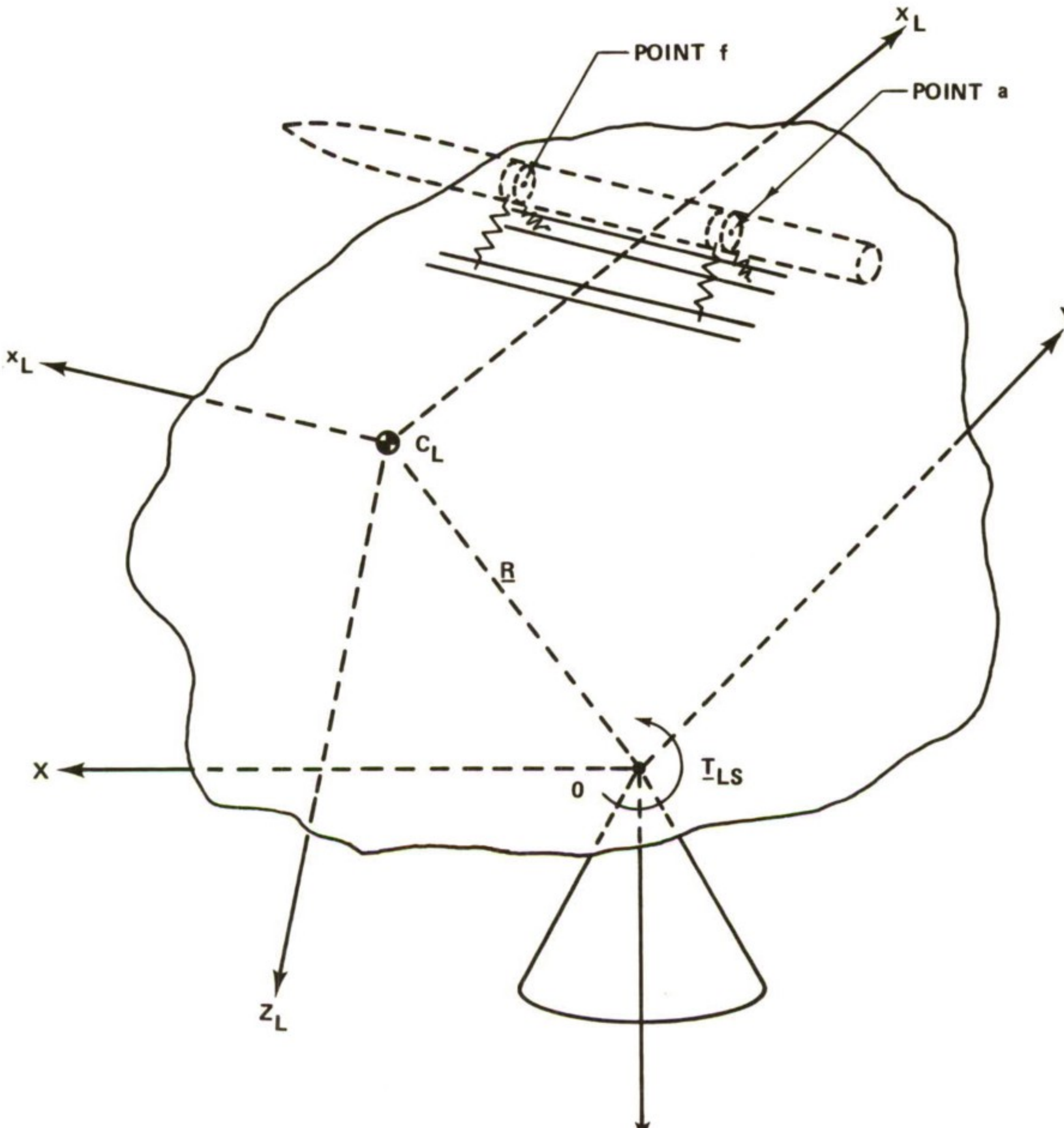

Figure 2. Launcher physical model.

directed parallel to the horizontal projection of the desired flight path of the rocket, and the $C_L x_L y_L z_L$ system, which is fixed in the launcher, are also shown in Figure 2. The stiffness and damping of the launcher are assumed to be concentrated at the point 0, exerting a torque, $\underline{T}_{LS}$, about the point. The mass distribution of the launcher may be arbitrary, but is assumed to be known.

The springs, which are shown connecting a rocket to the launcher, are intended to model the flexibility characteristics of the parts of the launcher and the rocket which couple the two, i.e., the launcher/rocket interface. After detent release the springs translate parallel to the x_L-axis with the rocket and exert forces in the y_L- and z_L-directions at the points f and a. Also, frictional forces resulting from sliding contact of the shoes (or other connecting devices) are assumed to act, opposing the sliding motion. Finally, gravitational effects predicated on a flat earth model are included.

In Figure 3, the vector $\underline{R}$ locates the launcher's center of mass and $\underline{R}$ and $\underline{S}_a$ together locate the point a, which lies on the centerline (geometric) of the missile and the $y_L z_L$ plane which contains the aft shoes. Furthermore, $M\underline{g}$ is the launcher weight vector.

c. Rocket Model

The rocket model which was adopted for this study is shown in Figure 3. The rocket is assumed to be a single rigid body of constant mass, m, and the mass distribution of the rocket is assumed to be arbitrary. The forces which are considered to be acting on the missile are the thrust, the rocket's weight, forces transmitted by the springs which connect the rocket to the launcher, frictional forces, the detent force (which acts at a), and external forces, such as pressure forces due to rocket plume impingement on the launcher and aerodynamic forces. The torques which act on the rocket include those due to the above forces and the spin torque generated by a spin motor or other device and frictional torques in the spin bearings (or tube reactions in a tube-type launcher).*

In Figure 3, the vector $\underline{u} + \underline{S}_a + \underline{R}$ locates the rocket's center of mass, which does not necessarily lie on the rocket's geometric axis of symmetry. The $C_m x_m y_m z_m$ coordinate system is fixed in the rocket, and the vectors $\underline{\ell}_{fa}$ and $\underline{\ell}_F$ will be used in following sections in the derivation of equations of motion.

*A nonspinning rocket model with the shoes modeled as springs has been briefly studied in Reference 4.

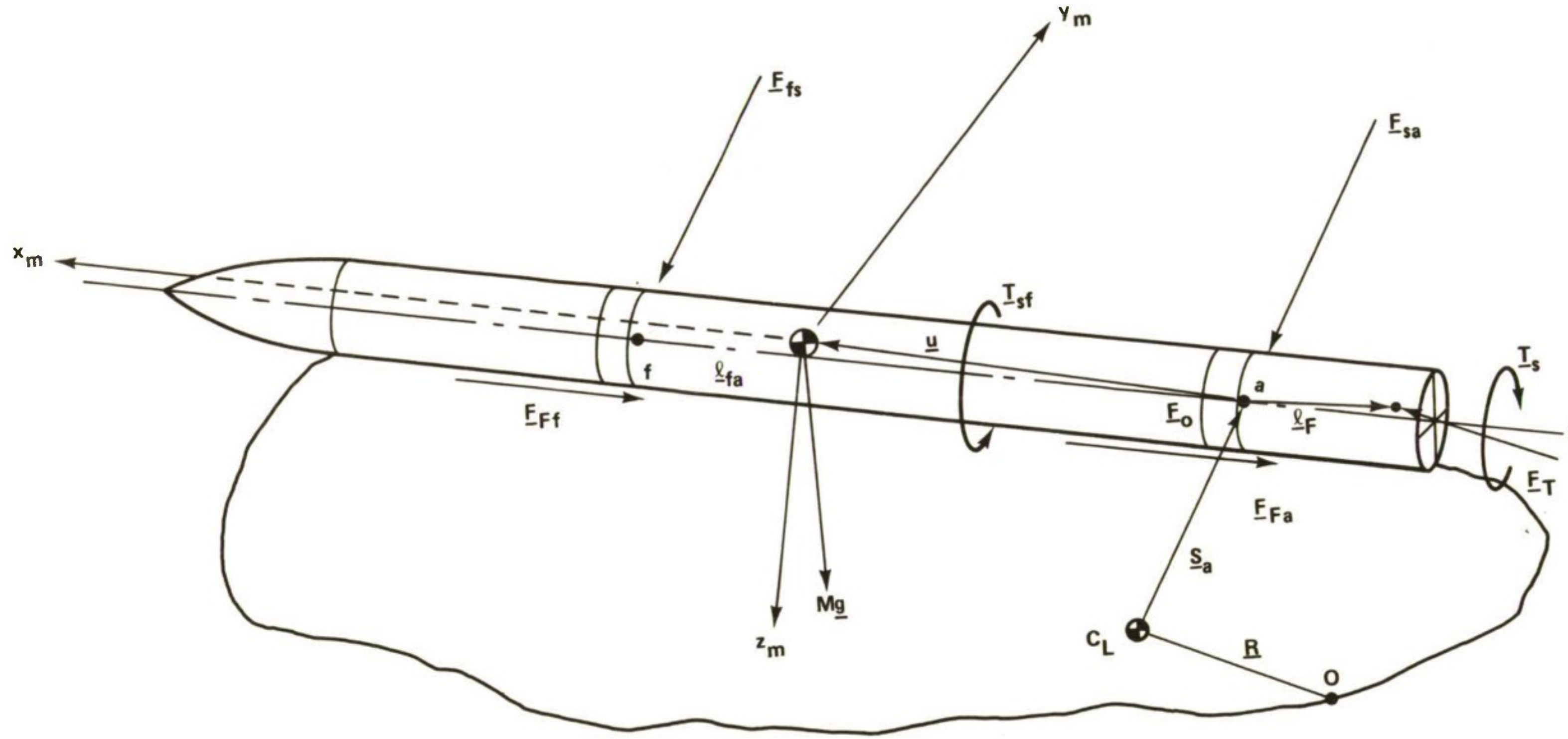

Figure 3. Rocket mathematical model.

Section III. EQUATIONS OF MOTION

a. General Comments

All derivations of equations of motion for dynamical systems in which relativistic effects may be ignored are based on Newton's laws of motion. Sometimes these laws are applied directly, an approach which we term "Eulerian." Other times the dynamicist may use the formalism of the Lagrangian and Hamiltonian* approaches in applying Newton's laws somewhat indirectly. The choice of approach, or method of attack, is somewhat arbitrary depending on the type of constraints, the purpose for which the equations are being obtained, the prejudices of the analyst, etc.

When the constraints on a fairly complex dynamical system are holonomic, Lagrange's equations are often used because this allows the constraints to be more or less automatically satisfied and because, once the kinetic energy and generalized forces have been formulated, the rest of the derivation is essentially mechanical.

The Lagrangian approach has drawbacks, however. For example, the second-order differential equations which are obtained thereby are often intricately coupled and very complicated and the algebra involved in the derivation, although mostly mechanical, may be extensive. Furthermore, if the system model is changed in some manner, it may be necessary to repeat a large portion of the derivation.

The Eulerian approach's major drawback is probably the difficulty of including the effects of constraints into the equations. However, the Eulerian approach is straightforward and, in the author's opinion, offers more flexibility than the Lagrangian approach, especially when modern vector/matrix methods are used to convert vector equations of motion to matrix equations which are to be solved numerically on a digital computer. Hence, the Eulerian approach was adopted for the derivation which follows in Subsections III.c through III.f.

As a prelude to the derivation, some comments on vector/matrix notation are made in Subsection III.b. Next, in Subsection III.c, Newton's laws are applied to obtain the governing dynamical equations in first-order form. The kinematical aspects of the launcher/rocket system are treated in Subsection III.d, while the effects of constraining the rocket's motion during the detent phase of launch are discussed in Subsection III.e and explicit expressions for the external forces and torques acting on the rocket and the system are determined in Subsection III.f.

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*Usually the Hamiltonian approach is reserved for use when the system is conservative.

b. Vector/Matrix Notation

In deriving equations of motion for complex dynamical systems, vectors, although not completely indispensable, are very nearly so. Vector differential equations cannot, of course, be directly programmed for numerical solution; however, one may transform such vector equations in equivalent matrix equations in which matrix operations isomorphic to vector operations, such as multiplying a row matrix times a column matrix to get the result of a vector dot product, are used. Moreover, by using matrix operators isomorphic to vector operators, much algebra may be avoided by the analyst and passed on to the computer.

As an example, we consider the vector equation of motion for a point mass m moving under the influence of an external force $\underline{F}$ in a coordinate system $0\epsilon\eta\zeta$ which has a fixed origin 0, but is rotating with angular velocity $\underline{\omega}$ and angular acceleration $\dot{\underline{\omega}}$ as shown in Figure 4.

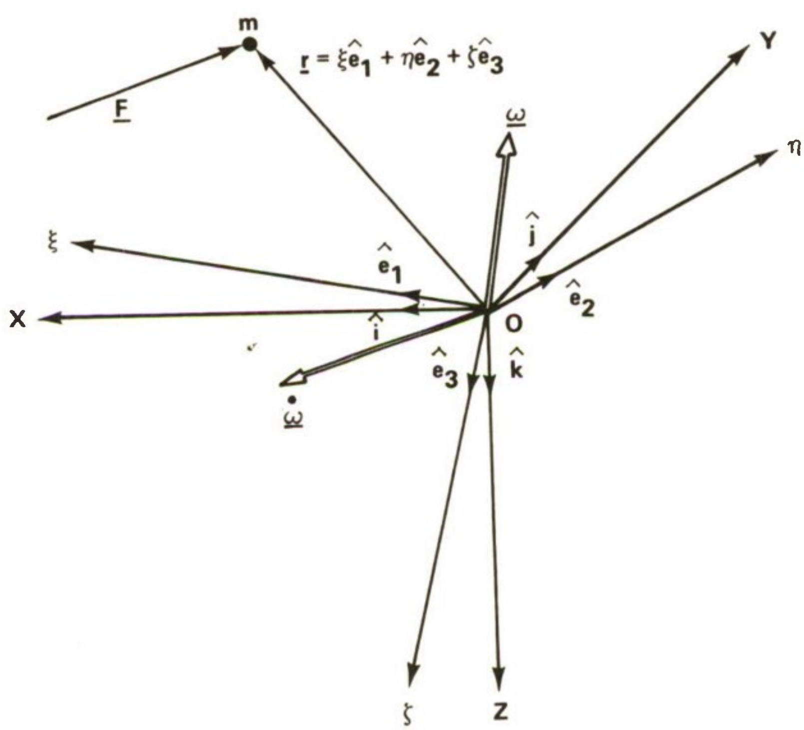

Figure 4. Geometry for the point mass motion example.

The equation of motion may be written in the vector form,

$$m\ddot{\underline{r}} = m[\mathring{\mathring{\underline{r}}} + \dot{\underline{\omega}} \times \underline{r} + 2\ \underline{\omega} \times \mathring{\underline{r}} + \underline{\omega} \times (\underline{\omega} \times \underline{r})] = \underline{F} \quad , \qquad (1)$$

where

$$\overset{\circ\circ}{\underline{r}} \equiv \ddot{\xi}\,\hat{e}_1 + \ddot{\eta}\hat{e}_2 + \ddot{\zeta}e_3 \quad , \tag{2}$$

$$\dot{\underline{\omega}} \equiv \dot{\omega}_1\hat{e}_1 + \dot{\omega}\hat{e}_2 + \dot{\omega}_3\hat{e}_3 \quad , \tag{3}$$

$$\overset{\circ}{\underline{r}} \equiv \dot{\xi}\hat{e}_1 + \dot{\eta}\hat{e}_2 + \dot{\xi}\hat{e}_3 \quad , \tag{4}$$

and the unit vectors $\hat{e}_j$ rotate with the $O\xi\eta\zeta$ system. Equation (1) holds regardless of the system in which $\underline{F}$ is expressed. For example, we may write $\underline{F}$ as

$$\underline{F} = F_X\hat{I} + F_Y\hat{J} + F_Z\hat{K} \quad , \tag{5}$$

where $\hat{I}$, $\hat{J}$, and $\hat{K}$ are nonrotating unit vectors associated with the inertial system OXYZ (Figure 4).

To write Equation (1) in an equivalent matrix form, we must first express each term in the same basis; i.e., each vector term must be expressed in terms of the same unit vectors. If $\underline{C}$ is the direction cosine matrix such that

$$\begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} = \underline{\underline{C}} \begin{pmatrix} \hat{I} \\ \hat{J} \\ \hat{K} \end{pmatrix} \quad , \tag{6}$$

then

$$\underline{F} = F_1\,\hat{e}_1 + F_2\,\hat{e}_2 + F_3\,\hat{e}_3 \quad , \tag{7}$$

where

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \underline{\underline{C}} \begin{pmatrix} F_X \\ F_Y \\ F_Z \end{pmatrix} \quad . \tag{8}$$

Using the same notation for column matrices and the vector counterparts, e.g.,

$$\underline{r} = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \quad \text{and} \quad \underline{F} = \begin{pmatrix} F_X \\ F_Y \\ F_Z \end{pmatrix} \quad , \tag{9}$$

we may write

$$m(\overset{\circ\circ}{\underline{r}} + \overset{\circ}{\tilde{\underline{\omega}}}\,\underline{r} + 2\tilde{\underline{\omega}}\,\overset{\circ}{\underline{r}} + \tilde{\underline{\omega}}\tilde{\underline{\omega}}\underline{r}) = \underline{\underline{C}}\;\underline{F} \quad , \tag{10}$$

where the tilde above a vector denotes a certain skew-symmetric matrix formed from its components. To illustrate the tilde notation, we consider [5]

$$\tilde{\underline{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad . \tag{11}$$

Upon multiplying $\tilde{\underline{\omega}}$ times the column matrix $\underline{r}$, we find that the elements of the product column vector are the components of $\underline{\omega}\mathrm{x}\underline{r}$ in the $(\hat{i}, \hat{j}, \hat{k})$ basis.

Returning to Equation (10), solving it for $\underline{r}$ and defining $(u\,v\,w)^T = \overset{\circ}{\underline{r}}$ (where T denotes the transpose), we obtain

$$(\dot{u}\;\;\dot{v}\;\;\dot{w})^T = -(\overset{\circ}{\tilde{\underline{\omega}}}\,\underline{r} + 2\tilde{\underline{\omega}}\underline{V} + \tilde{\underline{\omega}}\tilde{\underline{\omega}}\underline{r}) + \frac{1}{m}\,\underline{\underline{C}}\;\underline{F} \tag{12}$$

and, of course,

$$\overset{\circ}{\underline{r}} = (u\;\;v\;\;w)^T \tag{13}$$

as the equations which govern the motion of m (assuming that $\underline{\omega}$ and $\overset{\circ}{\underline{\omega}}$ are known).

The usefulness of vector-matrix conversion is now evident. General digital computer subroutines can be written to form tilde matrices from vectors and multiply matrices and hence evaluate the right-hand side of Equation (12), which will be a column matrix. Thus, the equations of motion of a complicated dynamical system do not have to be completely reduced to scalar form by hand to program them.

The advantages of using a digital computer to perform a great deal of the algebra are many, but two of the foremost ones are that mistakes in algebra (especially minus signs) may be avoided and that the programming statements may be more concise, hence avoiding programming errors.

In what follows, we will use the tilde notation just explained as well as (1) an underline for either a vector or a column matrix, (2) a double underline for a second-order tensor or a square matrix, (3) a superscript T to denote the transpose of a matrix, and (4) a superscript -1 to denote the inverse of a square matrix. Three other convenient definitions which will be utilized are

$$\underline{\underline{E}} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \tag{14}$$

$$\underline{\underline{E}}_1 \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \tag{15}$$

and

$$E_{23} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{16}$$

c. Dynamics

The approach used in obtaining the equations of motion of the launcher/rocket system is as follows. First, the vector equation governing the rotational motion of the system about 0 is obtained. When the rocket separates from the launcher, certain terms may be dropped from this equation to obtain the governing equation for the launcher alone. Second, the equations for rotation and translation of the rocket are developed. Two forms of the equation of rotational motion of the rocket are developed, since during detent and tipoff it is convenient to sum moments about the point a on the rocket centerline (Figure 2).

(1) System Rotational Motion. Consider Figure 5 in which the system of launcher plus rocket is depicted as two rigid bodies. Figure 5 shows the vectors $\underline{R}$, $\underline{S}_a$, and $\underline{u}$, which serve to locate the centers of mass

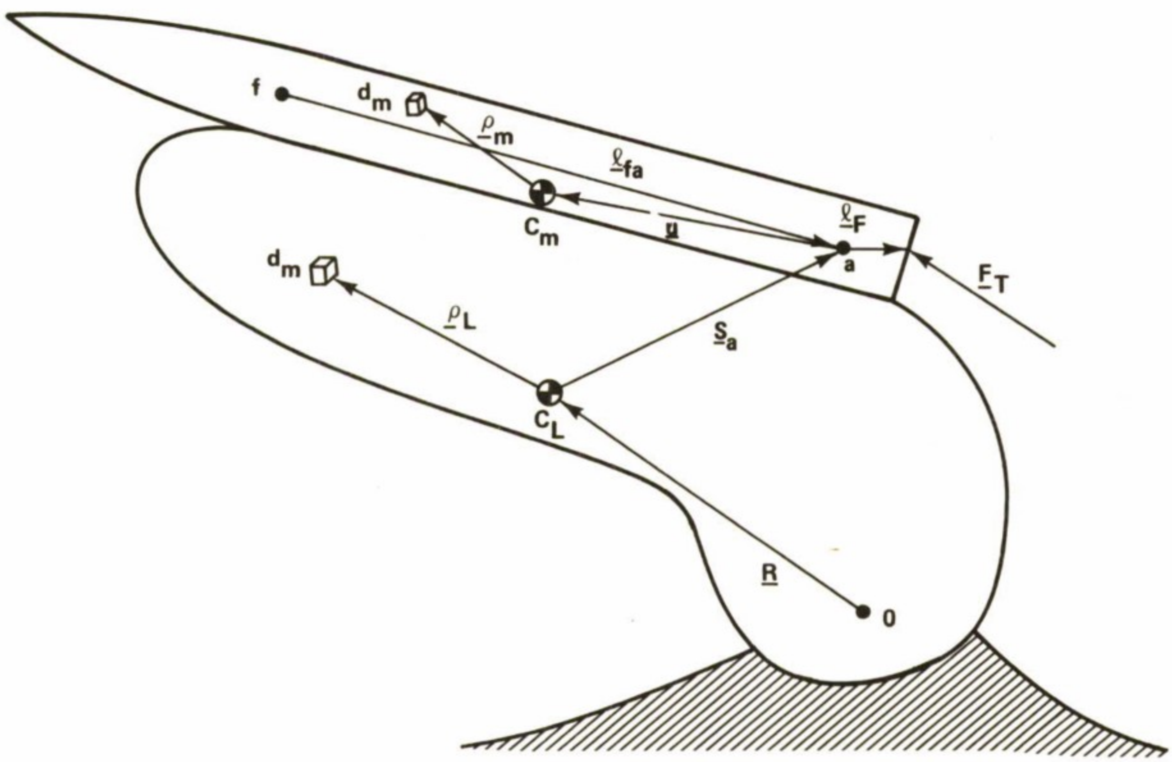

Figure 5. Geometric description of position vectors.

of the two bodies, and the generic position vectors $\underline{\rho}_L$ and $\underline{\rho}_m$ to elements of mass in the launcher and rocket, respectively, from their mass centers.

By definition, the angular momentum of the system about 0 is

$$\underline{H}_o = \int_M (\underline{R} + \underline{\rho}_L) \times (\dot{\underline{R}} + \dot{\underline{\rho}}_L)\, dM + \int_m (\underline{R} + \underline{S}_a + \underline{u} + \underline{\rho}_m)$$

$$\times\ (\dot{\underline{R}} + \dot{\underline{S}}_a + \dot{\underline{u}} + \dot{\underline{\rho}}_m)\, dm \quad . \tag{17}$$

The expression for $\underline{H}_o$ may be simplified by using the definition of the center of mass of a rigid body. We have

$$\int_M \underline{\rho}_L\, dM = \int_M \dot{\underline{\rho}}_L\, dM = \underline{0} \tag{18}$$

and

$$\int_m \underline{\rho}_m \, dm = \int_m \dot{\underline{\rho}}_m \, dm = \underline{0} \tag{19}$$

and, since $\underline{R}$, $\dot{\underline{R}}$, $\underline{S}_a$, $\dot{\underline{S}}_a$, $\underline{u}$, and $\dot{\underline{u}}$ do not vary during the indicated integration over the masses, we may simplify Equation (17) to obtain Equation (20):

$$\underline{H}_o = M\,\underline{R} \times \dot{\underline{R}} + \int_M \underline{\rho}_L \times \dot{\underline{\rho}}_L \, dM + m(\underline{R} + \underline{S}_a + \underline{u})$$

$$\times (\dot{\underline{R}} + \dot{\underline{S}}_a + \dot{\underline{u}}) + \int_m \underline{\rho}_m \times \dot{\underline{\rho}}_m \, dm \quad . \tag{20}$$

Let $\underline{\omega}$ and $\underline{\Omega}$ denote the angular velocities of the launcher and rocket, respectively, so that

$$\dot{\underline{\rho}}_L = \underline{\omega} \times \underline{\rho}_L \tag{21}$$

and

$$\dot{\underline{\rho}}_m = \underline{\Omega} \times \underline{\rho}_m \quad . \tag{22}$$

By substituting Equations (21) and (22) into Equation (20), it may be shown that

$$\underline{H}_o = \underline{\underline{I}}_{L/o} \cdot \underline{\omega} + m(\underline{R} + \underline{S}_a + \underline{u}) \times (\dot{\underline{R}} + \dot{\underline{S}}_a + \dot{\underline{u}}) + \underline{\underline{I}}_m \cdot \underline{\Omega} \quad , \tag{23}$$

where $\underline{\underline{I}}_{L/o}$ and $\underline{\underline{I}}_m$ are the inertia dyadics of the launcher with respect to point 0 and the rocket with respect to its center of mass C_m.

The time rate of change of $\underline{H}_o$, $\dot{\underline{H}}_o$, is equal to the external torque about 0, $\underline{T}_o$. Explicitly,

$$\dot{\underline{H}} = \underline{\underline{I}}_{L/o} \cdot \overset{\circ}{\underline{\omega}}_L + \underline{\omega} \times \underline{\underline{I}}_{L/o} \cdot \underline{\omega} + m(\underline{R} + \underline{S}_a + \underline{u})$$

$$\times (\ddot{\underline{R}} + \ddot{\underline{S}}_a + \ddot{\underline{u}}) + \underline{\underline{I}}_m \cdot \overset{\circ}{\underline{\Omega}}_m + \underline{\Omega} \times \underline{\underline{I}}_m \cdot \underline{\Omega} \quad , \tag{24}$$

where we have assumed that $\underline{\underline{I}}_{L/o}$ and $\underline{\omega}$ are expressed in the launcher basis and $\underline{\underline{I}}_m$ and $\underline{\Omega}$ in the rocket basis.* Equation (24) is the basic equation governing the rotational motion of the system about 0. We must, however, consider the rotation and translation of the rocket before we can solve Equation (24) for $\overset{\circ}{\underset{L}{\underline{\omega}}}$ (in matrix form), since $\overset{\circ}{\underset{L}{\underline{\omega}}}$ may be present implicitly in the third term during the detent phase. We also point out that in Equation (24) and in certain equations which follow a small circle above a vector and an L or m underneath it indicates the derivative of the components only of such a vector expressed in the launcher (L) or rocket (m) basis.

(2) <u>Rocket Rotational and Translational Motion</u>. Considering the rocket only, its equation of rotational motion may be expressed as

$$\underline{\underline{I}}_m \cdot \overset{\circ}{\underset{m}{\underline{\Omega}}} + \underline{\Omega} \times \underline{\underline{I}}_m \cdot \underline{\Omega} = \underline{T}_{m/cm} \quad , \tag{25}$$

or as

$$\underline{\underline{I}}_m \; \overset{\circ}{\underset{m}{\underline{\Omega}}} + \underline{\Omega} \times \underline{\underline{I}} \cdot \underline{\Omega} = -\underline{u} \times m(\ddot{\underline{R}} + \ddot{\underline{S}}_a + \ddot{\underline{u}}) + \underline{T}_{m/a} \quad , \tag{26}$$

where $\underline{T}_{m/C_m}$ and $\underline{T}_{m/a}$ are the torques on the rocket about its center of mass and about a, respectively.

The translation of the rocket's center of mass is governed by the equation,

$$m(\ddot{\underline{R}} + \ddot{\underline{S}}_a + \ddot{\underline{u}}) = \underline{F}_m \quad , \tag{27}$$

where $\underline{F}_m$ is the external force on the rocket. More explicitly, we have

*That is, $\underline{\underline{I}}_{L/o} = I_{x_L x_L} \hat{i}_L \hat{i}_L - I_{x_L y_L} \hat{i}_L \hat{j}_L - I_{x_L z_L} \hat{i}\,\hat{k} - I_{x_L y_L} \hat{j}_L \hat{k}_L$

$+ I_{y_L y_L} \hat{j}_L \hat{j}_L - I_{y_L z_L} \hat{j}_L \hat{k}_L - I_{x_L z_L} \hat{k}_L \hat{i}_L - I_{y_L z_L} \hat{k}_L \hat{j}_L + I_{z_L z_L} \hat{k}_L \hat{k}_L$ and

$\underline{\omega} = \omega_1 \hat{i}_L + \omega_2 \hat{j}_L + \omega_3 \hat{k}_L$, with similar definitions for $\underline{\underline{I}}_m$ and $\underline{\Omega}$.

$$
m\Big[\underline{\mathring{\omega}} \times \underline{R} + \underline{\omega} \times (\underline{\omega} \times \underline{R}) + \underset{L}{\underline{\overset{\circ\circ}{S}_a}} + 2\,\underline{\omega} \times \underset{L}{\underline{\mathring{S}_a}} + \underset{L}{\underline{\mathring{\omega}}} \times \underline{S}_a
$$

$$
+ \underline{\omega} \times (\underline{\omega} \times \underline{S}_a) + \underset{m}{\underline{\mathring{\Omega}}} \times \underline{u} + \underline{\Omega} \times (\underline{\Omega} \times \underline{u})\Big] = \underline{F}_m \quad . \tag{28}
$$

Equations (24) through (28) may be used in a variety of ways. What we really want to achieve, of course, is a consistent set of equations which may be solved for $\underset{L}{\underline{\mathring{\omega}}}$, $\underset{m}{\underline{\mathring{\Omega}}}$, and $\underset{L}{\underline{\overset{\circ\circ}{S}_a}}$.

Using Equations (26) and (27), Equation (24) can be rewritten in the form,

$$
\underline{\underline{I}}_{L/o} \cdot \underset{L}{\underline{\mathring{\omega}}} + \underline{\omega} \times \underline{\underline{I}}_{L/o} \cdot \underline{\omega} + (\underline{R} + \underline{S}_a + \underline{u}) \times \underline{F}_m + \underline{T}_{m/cm} = \underline{T}_o \quad , \tag{29}
$$

which is a suitable equation for $\underline{\mathring{\omega}}$, if $\underline{F}_m$ and $\underline{T}_{m/C_m}$ do not include constraint forces and torques; whereas, if they do, $\underline{\mathring{\omega}}$ will also appear in those forces and torques. We shall consider constraints on the rocket and their effects on the forms of the equations of motion in Subsection III.f.

Before considering the kinematical aspects of our problem, we note that $|\underline{\omega}|$ should be very small for most launchers so that we may drop the term $\underline{\omega} \times \underline{\underline{I}}_{L/o} \cdot \underline{\omega}$ from Equation (29) and similar terms which appear in Equation (28).*

d. <u>Kinematics</u>

Kinematics is a subject area which deals with the problem of describing position, velocity and acceleration in terms of geometric variables. In our case, we will use kinematical principles to (1) express $\underline{\omega}$ and $\underline{\Omega}$ in terms of Eulerian angles and their derivatives, (2) obtain an alternate equation of motion for the rocket's center of mass, (3) derive equations for the time rates of change of X, Y, and Z, the inertial coordinates of C_m, (4) obtain equations defining the orientation of the rocket with respect to the launcher, and (5) develop the transformation matrices needed to convert the dynamical (and some kinematical) equations to the $(\hat{i},\hat{j},\hat{k})$ (inertial), $(\hat{i}_L,\hat{j}_L,\hat{k}_L)$, or $(\hat{i}_m,\hat{j}_m,\hat{k}_m)$ basis.

*These terms have been retained in the computer code.

(1) Launcher Orientation and Angular Velocity. The orientation of the launcher may be defined by the set of Eulerian angles (α_2, α_3,α_1) shown in Figure 6. The rotation sequence of 2-3-1 was chosen so

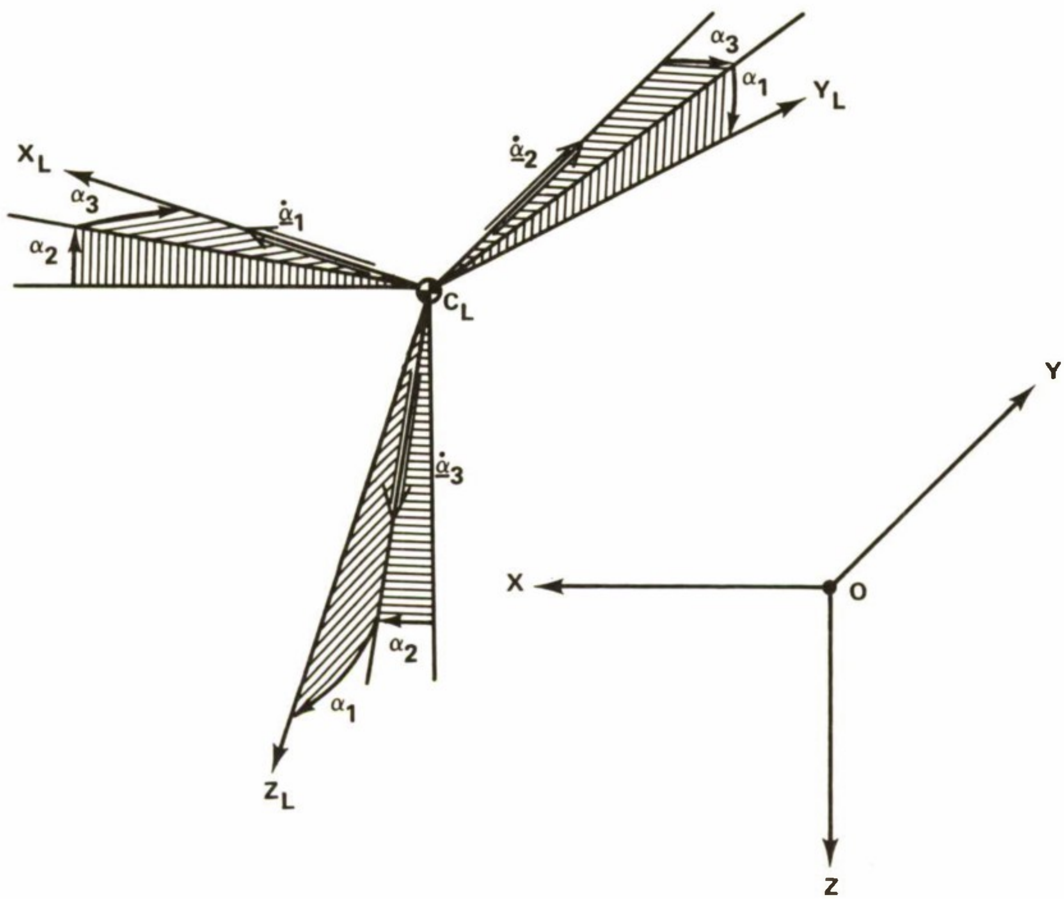

Figure 6. Launcher orientation.

that large values of the elevation angle α_2 may be easily incorporated. It is assumed, however, that α_3 and α_1 are small angles so that $\cos \alpha_3 \simeq \cos \alpha_1 \simeq 1$, $\sin \alpha_3 \simeq \alpha_3$, and $\sin \alpha_1 \simeq \alpha_1$. With this assumption, the direction cosine matrix which relates the $(\hat{I},\hat{J},\hat{K})$ and $(\hat{i}_L,\hat{j}_L,\hat{k}_L)$ bases; i.e., $(\hat{i}_L\hat{j}_L\hat{k}_L)^T = \underline{\underline{B}}(\hat{I}\ \hat{J}\ \hat{K})^T$ is

$$\underline{\underline{B}} \simeq \begin{bmatrix} C\alpha_2 & \alpha_3 & -S\alpha_2 \\ \alpha_1 S\alpha_2 - \alpha_3 C\alpha_2 & 1 & \alpha_3 S\alpha_2 + \dot{\alpha}_1 \alpha C\alpha_2 \\ S\alpha_2 & -\alpha_1 & C\alpha_2 \end{bmatrix} \tag{30}$$

where the shorthand notation $S \equiv \sin$ and $C \equiv \cos$ has been used.

The components of $\underline{\omega} = \omega_1 \hat{i}_L + \omega_2 \hat{j}_L + \omega_3 \hat{k}_L$, may be expressed in the forms,

$$\omega_1 = \dot{\alpha}_1 - \dot{\alpha}_2 \, S\alpha_3 \quad , \tag{31a}$$

$$\omega_2 = \dot{\alpha}_2 \, C\alpha_3 C\alpha_1 + \dot{\alpha}_3 S\alpha_1 \quad , \tag{31b}$$

and

$$\omega_3 = -\dot{\alpha}_2 C\alpha_3 S\alpha_1 + \dot{\alpha}_3 C\alpha_1 \quad . \tag{31c}$$

Since the $\dot{\alpha}_j$ are required to be small and α_2 and α_3 are small also,

$$\omega_1 \simeq \dot{\alpha}_1 \quad , \tag{32a}$$

$$\omega_2 \simeq \dot{\alpha}_2 \tag{32b}$$

and

$$\omega_3 \simeq \dot{\alpha}_3 \quad , \tag{32c}$$

or

$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \quad . \tag{33}$$

(2) Rocket Orientation and Angular Velocity. Because the rocket is assumed to be connected to the launcher by springs, the orientation of the rocket relative to the launcher as well as its

inertial orientation should be known. We also require knowledge of the rocket's angular velocity relative to the launcher, since the springs possess structural damping characteristics.

The inertial attitude of the rocket may be defined by a 3-2-1 sequence of Eulerian angles denoted by ψ, θ, and ϕ, respectively, and displayed in Figure 7. The matrix $\underline{\underline{C}}$ which defines the transformation

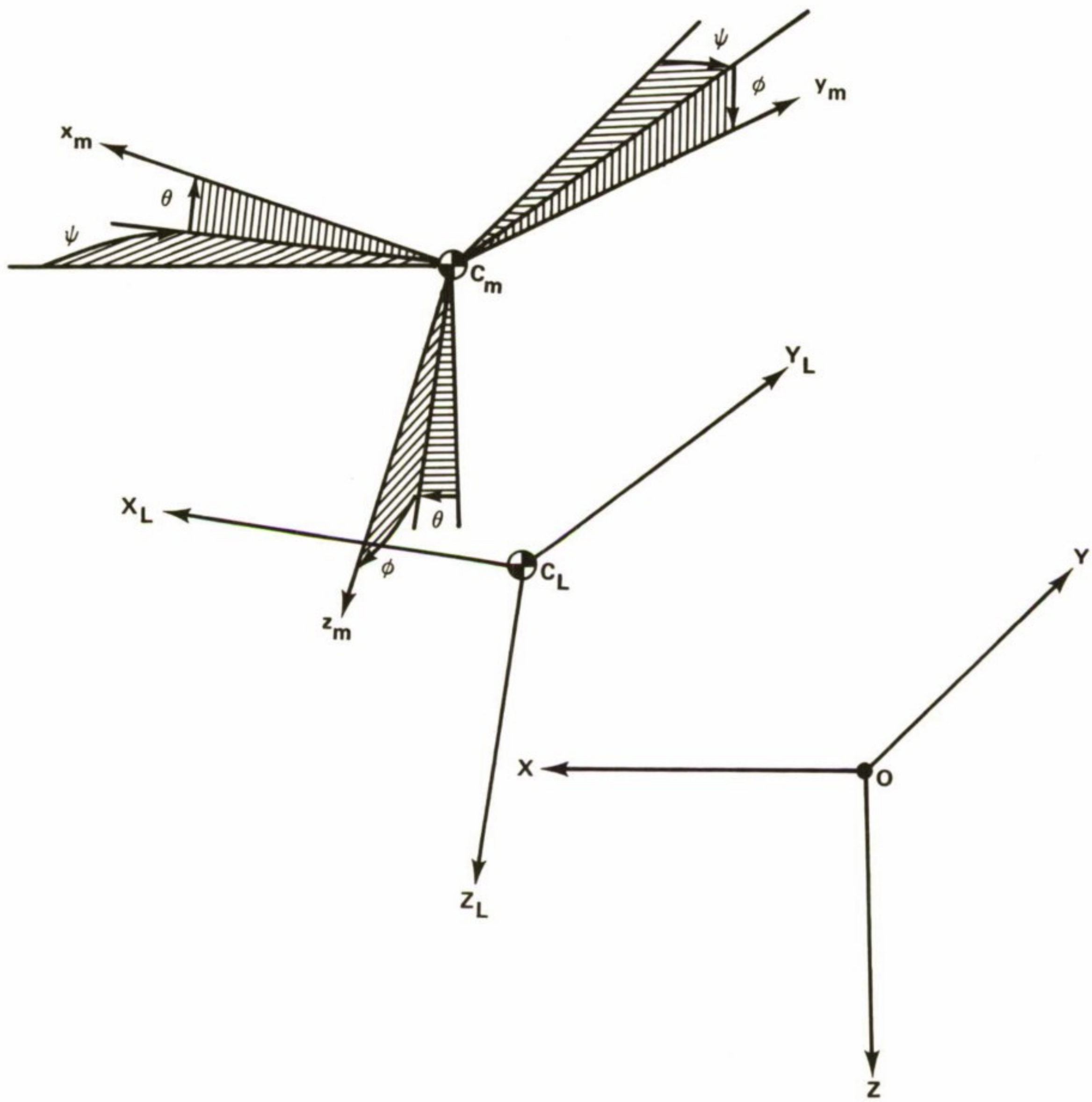

Figure 7. Inertial orientation of the rocket.

from the $(\hat{I}\ \hat{J}\ \hat{K})$ basis to the $(\hat{i}\ \hat{j}\ \hat{k})$ basis is, in rotation matrix form,

$$\underline{\underline{C}} = \begin{bmatrix} 1 & 0. & 0 \\ 0 & C\Phi & S\Phi \\ 0 & -S\Phi & C\Phi \end{bmatrix} \begin{bmatrix} C\theta & \theta & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} C\psi & S\psi & 0 \\ -S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{34}$$

and the components of $\underline{\Omega} = \Omega_1 \hat{i}_m + \Omega_2 \hat{j}_m + \Omega_3 \hat{k}_m$ may be expressed in the forms,

$$\Omega_1 = \dot{\Phi} - \dot{\psi}\ S\theta \quad , \tag{35a}$$

$$\Omega_2 = \dot{\theta}\ C\Phi + \dot{\psi}\ C\theta\ S\Phi \quad , \tag{35b}$$

and

$$\Omega_3 = -\dot{\theta}\ S\Phi + \dot{\psi}\ C\theta\ C\Phi \quad . \tag{35c}$$

Equation (35) may be used to obtain

$$\dot{\Phi} = \Omega_1 + (\Omega_2 S\Phi + \Omega_3 C\Phi)\ \tan\theta \quad , \tag{36a}$$

$$\dot{\theta} = \Omega_2\ C\Phi - \Omega_3 S\ \Phi \quad , \tag{36b}$$

$$\dot{\psi} = \frac{\Omega_2 S\Phi + \Omega_3\ C\Phi}{C\theta} \tag{36c}$$

Since only the angle ψ may be assumed to be small, Equation (36) may not be simplified.

The rocket's orientation relative to the launcher-fixed $C_L x_L y_L z_L$ system may be defined by the Eulerian angle set $(\theta_3, \theta_2, \theta_1)$. These angles are depicted in Figure 8. The rotation sequence is the same as that for the set (ψ, θ, Φ) but both θ_3 and θ_2 may be assumed to be small. By using the angles θ_j we may form a matrix $\underline{\underline{A}}$ which specifies the transformation from the $(\hat{i}_L, \hat{j}_L, \hat{k}_L)$ basis to the $(\hat{i}_m\ \hat{j}_m\ \hat{k}_m)$ basis. Explicitly,

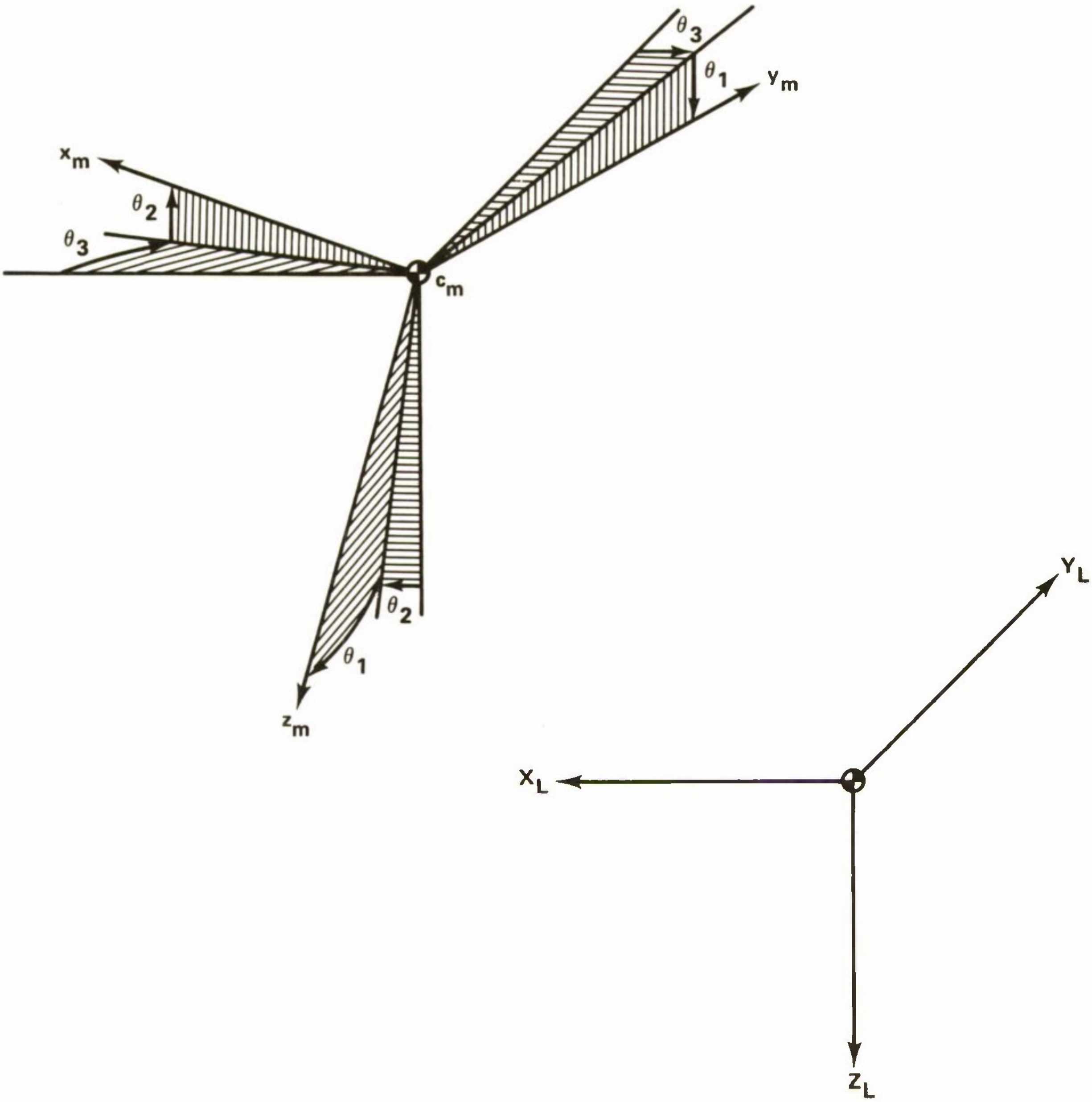

Figure 8. Orientation of the rocket relative to the launcher.

$$\underline{\underline{A}} \simeq \begin{bmatrix} 1 & \theta_3 & -\theta_2 \\ -\theta_3 C\theta_1 + \theta_2 S\theta_1 & C\theta_1 & S\theta_1 \\ \theta_3 S\theta_1 + \theta_2 C\theta_1 & -S\theta_1 & C\theta_1 \end{bmatrix} . \tag{37}$$

The angular rates $\dot{\theta}_j$ are related to $\underline{\Omega}$ and $\underline{\omega}$ by the equation,

$$\underline{\Omega} - \underline{\underline{A}}\,\underline{\omega} = \begin{bmatrix} \dot{\theta}_1 - \dot{\theta}_3\, S\dot{\theta}_2 \\ \dot{\theta}_2 C\theta_1 + \dot{\theta}_3 C\theta_2 S\theta_1 \\ -\dot{\theta}_2 S\theta_1 + \dot{\theta}_3\, C\theta_2\, C\theta_1 \end{bmatrix} \quad . \tag{38}$$

Since θ_2 and θ_3 are small,

$$\underline{\underline{A}}\,\underline{\omega} \simeq \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta_1 & S\theta_1 \\ 0 & -S\theta_1 & C\theta_1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \quad , \tag{39}$$

so that

$$\dot{\theta}_1 \simeq \Omega_1 + (\Omega_2 S\theta_1 + \Omega_2 C\theta_1 - \omega_3)\tan\,\theta_2 - \omega_1 \quad , \tag{40a}$$

$$\dot{\theta}_2 \simeq \Omega_2\, C\theta_1 - \Omega_3\, S\theta_1 - \omega_2 \quad , \tag{40b}$$

and

$$\dot{\theta}_3 \simeq \frac{\Omega_2\, S\theta_1 + \Omega_3\, C\theta_1 - \omega_3}{C\theta_2} \quad . \tag{40c}$$

Assuming that θ_2 is appropriately small,

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \simeq \begin{bmatrix} \Omega_1 - \omega_1 \\ \Omega_2 C\theta_1 - \Omega_3 S\theta_1 - \omega_2 \\ \Omega_2 S\theta_1 + \Omega_3 C\theta_1 - \omega_3 \end{bmatrix} \quad . \tag{41}$$

Equations (30), (33), (34), (36), (37), and (41) are sufficient to connect the rotational motions of the system with the physical world through geometry. We still need analogous equations for the translational motion of the rocket.

Let the velocity $\underline{V}$ of C_m be expressed as

$$\underline{V} = U\,\hat{i}_m + V\,\hat{j}_m + W\,\hat{k}_m \quad . \tag{42}$$

We then may write

$$\dot{\underline{V}} = \mathring{\underline{V}} + \underline{\Omega} \times \underline{V} \quad . \tag{43}$$

Since

$$\dot{\underline{V}} = \frac{\underline{F}_m}{m} \quad , \tag{44}$$

$$\mathring{\underline{V}} = \underline{\Omega} \times \underline{V} + \frac{\underline{F}_m}{m} \quad . \tag{45}$$

A matrix form of Equation (45) is

$$\begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix} = -\underline{\underline{\Omega}}\ \underline{V} + \underline{\underline{A}}\ \frac{\underline{F}_m}{m} \quad , \tag{46}$$

if we assume that $\underline{F}_m$ is expressed in the $(\hat{i}_L, \hat{j}_L, \hat{k}_L)$ basis.

The velocity of C_m may also be expressed in the form,

$$\underline{V} = \dot{X}\,\hat{I} + \dot{Y}\,\hat{J} + \dot{Z}\,\hat{K} \quad , \tag{47}$$

and it then follows from the definitions of $\underline{\underline{A}}$, $\underline{\underline{B}}$, and $\underline{\underline{C}}$ that

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \underline{\underline{B}}^T\ \underline{\underline{A}}^T \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \underline{\underline{C}}^T \begin{bmatrix} U \\ V \\ W \end{bmatrix} \tag{48}$$

Because we also want to know the vector $\underline{S}_a$, we let

$$\underline{S}_a = X_{L_a}\,\hat{i}_L + Y_{L_a}\,\hat{j}_L + Z_{L_a}\,\hat{k}_L \quad , \tag{49}$$

$$X_{L_a} = X_{L_{a_o}} + \delta_1 \quad , \tag{50a}$$

$$Y_{L_a} = Y_{L_{a_o}} + \delta_2 \quad , \tag{50b}$$

and

$$Z_{L_a} = Z_{L_{a_o}} + \delta_3 \quad , \tag{50c}$$

where $X_{L_{a_o}}$, $Y_{L_{a_o}}$, $Z_{L_{a_o}}$ are initial values of the indicated variables, and consider Equation (28). We thereby obtain the matrix equation,

$$\begin{bmatrix} \ddot{\delta}_1 \\ \ddot{\delta}_2 \\ \ddot{\delta}_3 \end{bmatrix} = \frac{\underline{F}_m}{m} - \underline{\tilde{\mathring{\omega}}}_L \left(\underline{R} + \underline{S}_a\right) - 2\, \underline{\tilde{\omega}} \left(\dot{\delta}_1 \dot{\delta}_2 \dot{\delta}_3\right)^T - \underline{\underline{A}}^T \underline{\tilde{\mathring{\Omega}}}_m\, \underline{u} - \underline{\underline{A}}^T \underline{\tilde{\Omega}}\, \underline{\tilde{\Omega}}\, \underline{u} \quad . \tag{51}$$

We now have dynamical and kinematical equations for the system when there are no constraints, since Equations (25) and (29) are then uncoupled as far as $\underline{\mathring{\omega}}$ amd $\underline{\mathring{\Omega}}$ are concerned. Thus, Equations (25) and (29) can be solved for the matrix equivalents of these vectors and the results substituted into Equation (51) when there is no detent constraint. However, when the rocket is constrained by the detent force, which we assume acts through the point a, Equations (25) and (29) are, in general, coupled, and equations which account for this fact must be obtained for that phase of the launch. We also need to convert the remaining equations for rotational motion into appropriate matrix equations and to obtain expressions for the forces and moments. These things are done in the next two subsections.

e. <u>Constraints and Matrix Forms of the Equations</u>

The detent force is assumed to be the only "hard" constraint on the rocket's motion; i.e., it is the only constraint that completely prohibits motion. This force, which, of course, varies in magnitude and direction during spinup and thrust buildup, is, as stated, assumed to act through the point a, and to prohibit motion of the rocket in the x_L direction. This constraint may be defined mathematically as

$$\ddot{x}_{L_a} = 0 \tag{52}$$

and

$$\dot{x}_{L_a} = 0 \tag{53}$$

during detent. If we let

$$\underline{F}_m = \underline{F}_{m_{ex}} + \underline{F}_{m_D} + \underline{F}_{m_F} \quad ,$$

where $\underline{F}_{m_{ex}}$ is the resultant of the external forces on the rocket (including those due to gravity, thrust and the rocket springs), $\underline{F}_{m_D}$ is the detent force and $\underline{F}_{m_F}$ is the frictional force, then Equations (52) and (53) are satisfied if

$$\Big[\underline{F}_{m_{ex}} - 2\,m\,\underline{\omega} \times \left(\dot{\delta}_2\,\hat{j}_L + \dot{\delta}_3\,\hat{k}_L\right) - m\underline{\Omega} \times (\underline{\Omega} \times \underline{u})$$

$$+ m\underline{u} \times \mathring{\underline{\Omega}} + m(\underline{R} + \underline{S}_a) \times \mathring{\underline{\omega}}_L - \underline{F}_{m_D} + \underline{F}_{m_F}\Big] \cdot \hat{i}_L = 0 \quad . \tag{54}$$

The detent force has only an x_L-component in our model, thus

$$\underline{F}_{m_D} = \Big\{\Big[2\,m\,\underline{\omega} \times \left(\dot{\delta}_2\,\hat{j}_L + \dot{\delta}_3\,\hat{k}_L\right) + m\,\underline{\Omega} \times (\underline{\Omega} \times \underline{u})$$

$$- m(\underline{R} + \underline{S}_a) \times \mathring{\underline{\omega}}_L - m\,\underline{u} \times \mathring{\underline{\Omega}}_m - \underline{F}_{m_{ex}} - \underline{F}_{m_F}\Big] \cdot \hat{i}_L\Big\}\,\hat{i}_L \quad . \tag{55}$$

The rocket will start to move in the x_L-direction when

$$\Big|\underline{F}_{m_{ex}} - 2\,m\,\underline{\omega} \times (\dot{\delta}_2\,\hat{j}_L + \dot{\delta}_3\,\hat{k}) - m\,\underline{\Omega} \times (\underline{\Omega} \times \underline{u})$$

$$+ m(\underline{R} + \underline{S}_a) \times \mathring{\underline{\omega}}_L + m\,\underline{u} \times \underline{\Omega}_m\Big| > \left|\underline{F}_{m_{D_{max}}}\right| + \left|\underline{F}_{m_F}\right| \quad . \tag{56}$$

Inequality (56) may be used as a check to determine when detent release occurs.

Consider now Equations (26), (27), (54), and (55). Combining these equations, we obtain

$$
\begin{aligned}
&\underline{\underline{I}}_m \cdot \overset{\circ}{\underline{\Omega}}_m - \underline{u} \times \left[m(\underline{R} + \underline{S}_a) \times \overset{\circ}{\underline{\omega}}_L \cdot \hat{i}_L \right] \hat{i}_L \\
&- \underline{u} \times \left[m\, \underline{u} \times \overset{\circ}{\underline{\Omega}}_m \cdot \hat{i}_L \right] \hat{i}_L + \underline{\Omega} \times \underline{\underline{I}}_m \cdot \underline{\Omega} \\
&= -\underline{u} \times \left[\left(\underline{F}_{m_{ex}} \cdot \hat{j}_L \right) \hat{j}_L + \left(\underline{F}_{m_{ex}} \cdot \hat{k}_L \right) \hat{k}_L \right] \\
&- \underline{u} \times \left\{ 2_m \left[\underline{\omega} \times \left(\dot{\delta}_2\, \hat{j}_L + \dot{\delta}_3\, \hat{k}_L \right) \right] \cdot \hat{i}_L \right. \\
&\left. + m[\underline{\Omega} \times (\underline{\Omega} \times \underline{u})] \cdot \hat{j}_L \right\} \hat{i}_L + \underline{T}_{m/a} \, , \qquad (57)
\end{aligned}
$$

as the equation of rotational motion of the rocket before detent release. In a similar manner, we obtain for the launcher, the equation,

$$
\begin{aligned}
&\underline{\underline{I}}_{L/o} \cdot \overset{\circ}{\underline{\omega}}_L - (\underline{R} + \underline{S}_a) \times \left\{ m \left[(\underline{R} + \underline{S}_a) \times \overset{\circ}{\underline{\omega}}_L \right] \cdot \hat{i}_L \right\} \hat{i}_L - (\underline{R} + \underline{S}_a) \\
&\times \left[m \left(\underline{u} \times \overset{\circ}{\underline{\Omega}}_m \right) \cdot \hat{i}_L \right] \hat{i}_L = -(\underline{R} + \underline{S}_a) \times \left\{ 2m \left[\underline{\omega} \times \left(\dot{\delta}_2\, \hat{j}_L + \dot{\delta}_3\, \hat{k}_L \right) \right] \right. \\
&\left. \cdot\, \hat{i}_L + m[\underline{\Omega} \times (\underline{\Omega} \times \underline{u})] \cdot \hat{i}_L \right\} \hat{i}_L - (\underline{R} + \underline{S}_a) \times \left[\left(\underline{F}_{m_{ex}} \cdot \hat{j}_L \right) \hat{j}_L \right. \\
&\left. + \left(\underline{F}_{m_{ex}} \cdot \hat{k}_L \right) \hat{k}_L \right] + \underline{T}_o - \underline{T}_{m/a} \quad . \qquad (58)
\end{aligned}
$$

Equations (57) and (58) may be combined to form the matrix equation,

$$
\left[\begin{array}{c|c} \underline{\underline{J}}_{11} & \underline{\underline{J}}_{12} \\ \hline \underline{\underline{J}}_{21} & \underline{\underline{J}}_{22} \end{array} \right]
\left[\begin{array}{c} \overset{\circ}{\underline{\Omega}}_m \\ \hline \overset{\circ}{\underline{\omega}}_L \end{array} \right]
=
\left[\begin{array}{c}
-\underline{\underline{\tilde{u}}}\, \underline{\underline{A}}\, \underline{\underline{E}}_{23}\, \underline{F}_{m_{ex}} + \underline{T}_{m/a} - \underline{\Omega}\, \underline{\underline{I}}_m \cdot \underline{\Omega} \\
\hline
-(\underline{R} + \underline{S}_a)\, \underline{\underline{E}}_{23}\, \underline{F}_{m_{ex}} - \underline{\underline{A}}^T\, \underline{T}_{m/a} + \underline{T}_o
\end{array} \right.
$$

$$
\left. \begin{array}{c}
-2\, m\, \underline{\underline{\tilde{u}}}\, \underline{\underline{A}}\, \underline{\underline{E}}_1\, \underline{\omega}\, \underline{\underline{E}}_{23}\, \overset{\circ}{\underline{S}}_{a_L} - m\, \underline{\underline{\tilde{u}}}\, \underline{\underline{A}}\, \underline{\underline{E}}_1\, \underline{\underline{A}}^T\, \tilde{\Omega}\, \tilde{\Omega}\, \underline{u} \\
\hline
-2\, m\, (\underline{R} + \underline{S}_a)\, \underline{\underline{E}}_1\, \underline{\omega}\, \underline{\underline{E}}_{23}\, \overset{\circ}{\underline{S}}_{a_L} - m\, (\underline{R} + \underline{S}_a)\, \underline{\underline{E}}_1\, \underline{\underline{A}}^T\, \tilde{\Omega}\, \tilde{\Omega}\, \underline{u}
\end{array} \right] , \qquad (59)
$$

where we have assumed that $\underline{T}_{m/a}$ is expressed in the $(\hat{i}_m,\hat{j}_m,\hat{k}_m)$ basis and $\underline{T}_o$ and $\underline{F}_m$ in the $(\hat{i}_L,\hat{j}_L,\hat{k}_L)$ basis and where

$$\underline{\underline{J}}_{11} \equiv \underline{\underline{I}}_m - m \, \tilde{\underline{u}} \, \underline{\underline{A}} \, \underline{\underline{E}}_1 \, \underline{\underline{A}}^T \, \tilde{\underline{u}} \quad , \tag{60a}$$

$$\underline{\underline{J}}_{12} \equiv -m \, \tilde{\underline{u}} \, \underline{\underline{A}} \, \underline{\underline{E}}_1 \, \widetilde{(\underline{R} + \underline{S}_a)} \quad , \tag{60b}$$

$$\underline{\underline{J}}_{21} \equiv -m \, \widetilde{(\underline{R} + \underline{S}_a)} \, \underline{\underline{E}}_1 \, \underline{\underline{A}}^T \, \tilde{\underline{u}} = \underline{\underline{J}}_{12}^T \quad , \tag{60c}$$

and

$$\underline{\underline{J}}_{22} \equiv \underline{\underline{I}}_{L/o} - m \, \widetilde{(\underline{R} + \underline{S}_a)} \, \underline{\underline{E}}_1 \, \widetilde{(\underline{R} + \underline{S}_a)} \quad . \tag{60d}$$

Hence, by inverting the matrix,

$$\underline{\underline{J}} = \left[\begin{array}{c|c} \underline{\underline{J}}_{11} & \underline{\underline{J}}_{12} \\ \hline \underline{\underline{J}}_{21} & \underline{\underline{J}}_{22} \end{array}\right] \quad , \tag{61}$$

and premultiplying Equation (59) by $\underline{\underline{J}}$'s inverse, $\underline{\underline{L}}$, we get

$$\left[\begin{array}{c} \overset{\circ}{\underline{\Omega}} \\ \hline \overset{\circ}{\underline{\omega}} \end{array}\right] \left[\begin{array}{c} \underline{\underline{L}}_{11} \, \underline{T}_m + \underline{\underline{L}}_{12} \, \underline{T}_L \\ \hline \underline{\underline{L}}_{21} \, \underline{T}_m + \underline{\underline{L}}_{22} \, \underline{T}_L \end{array}\right] \quad , \tag{62}$$

where $\underline{T}_m$ and $\underline{T}_L$ are defined such that

$$\underline{\underline{J}} \left[\begin{array}{c} \overset{\circ}{\underline{\Omega}}_m \\ \hline \overset{\circ}{\underline{\omega}}_L \end{array}\right] = \left[\begin{array}{c} \underline{T}_m \\ \hline \underline{T}_L \end{array}\right] \quad . \tag{63}$$

The inverse of $\underline{\underline{J}}$ may be determined by inverting only 3 × 3 matrices, which can be easily done explicitly. The formulas for computing the $\underline{\underline{L}}_{ij}$ are [6]

$$\underline{\underline{L}}_{11} = \left(\underline{\underline{J}}_{11} - \underline{\underline{J}}_{12}\,\underline{\underline{J}}_{22}^{-1}\,\underline{\underline{J}}_{21}\right)^{-1} \quad , \tag{64a}$$

$$\underline{\underline{L}}_{22} = \left(\underline{\underline{J}}_{22} - \underline{\underline{J}}_{21}\,\underline{\underline{J}}_{11}^{-1}\,\underline{\underline{J}}_{12}\right)^{-1} \quad , \tag{64b}$$

$$\underline{\underline{L}}_{12} = -\underline{\underline{J}}_{11}^{-1}\,\underline{\underline{J}}_{12}\,\underline{\underline{L}}_{22} \quad , \tag{64c}$$

and

$$\underline{\underline{L}}_{21} = -\underline{\underline{J}}_{22}^{-1}\,\underline{\underline{J}}_{21}\,\underline{\underline{L}}_{11} \quad . \tag{64d}$$

The matrix equations of motion for post-detent phases are

$$\underline{\underline{I}}_{m}\,\overset{\circ}{\underline{\Omega}}_{m} = -\underline{\tilde{\Omega}}\,\underline{\underline{I}}_{m}\,\underline{\Omega} + T_{m/Cm} \tag{65a}$$

and

$$\underline{\underline{I}}_{L/o}\,\overset{\circ}{\underline{\omega}}_{L} = \underline{T}_{o} - \underline{\underline{A}}^{T}\,\underline{T}_{m/Cm} - \left[\widetilde{(\underline{R} + \underline{S}_{a})} + \underline{\underline{A}}^{T}\,\underline{\tilde{u}}\,\underline{\underline{A}}\right]\underline{F}_{m} \quad , \tag{65b}$$

where

$$\underline{F}_{m} = \underline{F}_{m_{ex}} + \underline{F}_{m_{F}} \quad .$$

Equations (30), (33), (36), (37), (41), (46), (48), (51), and (62) or (65) are those required to simulate the motion of the system. However, $\underline{F}_m$, $\underline{T}_{m/a}$, and $\underline{T}_o$ must be defined before these equations may be solved. Such definitions are given in the following subsection.

f. Forces, Torques, and Equilibrium

(1) Forces and Torques on the Rocket. The forces which act on the rocket were assumed, for the purposes of this study, to be (1) the force due to gravity, $\underline{F}_{m_g}$ (a flat earth is assumed), (2) the thrust force, $\underline{F}_{m_T}$, (3) the detent force, $\underline{F}_D$, which has previously been discussed, (4) a frictional force which opposes the rocket's motion parallel to the

x_L-axis, and (5) the forces $\underline{F}_{fs}$ and F_{as} in the fore and aft springs which couple the rocket to the launcher. Explicit expressions for these forces are derived in what follows.

(a) Gravity Force. The gravity force on the rocket is $mg\hat{K}$ so that in matrix form (in the $\hat{i}_L, \hat{j}_L, \hat{k}_L$ basis)

$$\underline{F}_{m_g} = \underline{\underline{B}} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} , \tag{66}$$

where g is of course the local acceleration of gravity.

(b) Thrust Force. The thrust force $\underline{F}_{m_T}$ is obtained by assuming that the thrust $\underline{F}_T$ is rocket-fixed and is misaligned with the x_m-axis only a small amount. As shown in Figure 9, the angular thrust malalignment angles are denoted by α_y (a rotation about a line parallel to the y_m-axis) and α_z (a rotation "essentially" about a line parallel to the z_m-axis.

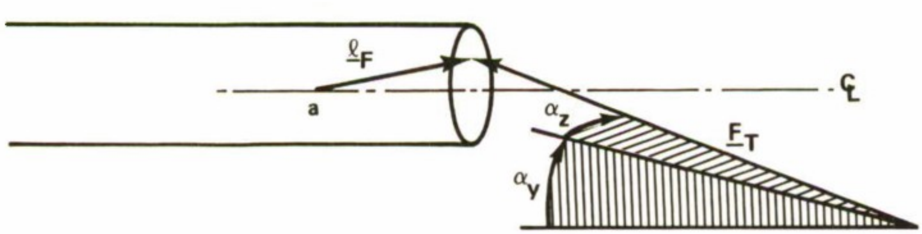

Figure 9. Thrust vector definition.

The magnitude of $\underline{F}_T$ is denoted F, so that,

$$\underline{F}_T \simeq F\,\hat{i}_m + \alpha_z F\,\hat{j}_m - \alpha_y F\,\hat{k}_m \quad . \tag{67}$$

From Equation (67), it follows that, in matrix form,

$$\underline{F}_{m_T} = \underline{\underline{A}}^T \underline{F}_T \quad . \tag{68}$$

(c) Spring Forces. The forces in the springs which couple the rocket to the launcher are assumed to be linear functions of the relative displacements and velocities in planes parallel to the $y_L z_L$-plane of the points f and a, measured, of course, from their undeformed positions in these planes. Thus, the force in the aft springs is (matrix form)

$$\underline{F}_{as} = -\underline{\underline{C}}_a \begin{bmatrix} 0 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \end{bmatrix} - K_a \begin{bmatrix} 0 \\ \delta_2 \\ \delta_3 \end{bmatrix} , \qquad (69)$$

where $\underline{\underline{C}}_a$ and $\underline{\underline{K}}_a$ are damping and stiffness matrices, respectively.

To determine the force in the forward springs, we consider Figure 10. From this figure, we conclude that

$$\underline{F}_{fs} = -\underline{\underline{C}}_f \left\{ \begin{bmatrix} 0 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \end{bmatrix} + \tilde{\underline{\ell}}_{fa} \begin{bmatrix} 0 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \right\} - \underline{\underline{K}}_f \left\{ \begin{bmatrix} 0 \\ \delta_2 \\ \delta_3 \end{bmatrix} + \tilde{\underline{\ell}}_{fa} \begin{bmatrix} 0 \\ \theta_2 \\ \theta_3 \end{bmatrix} \right\} , \qquad (70)$$

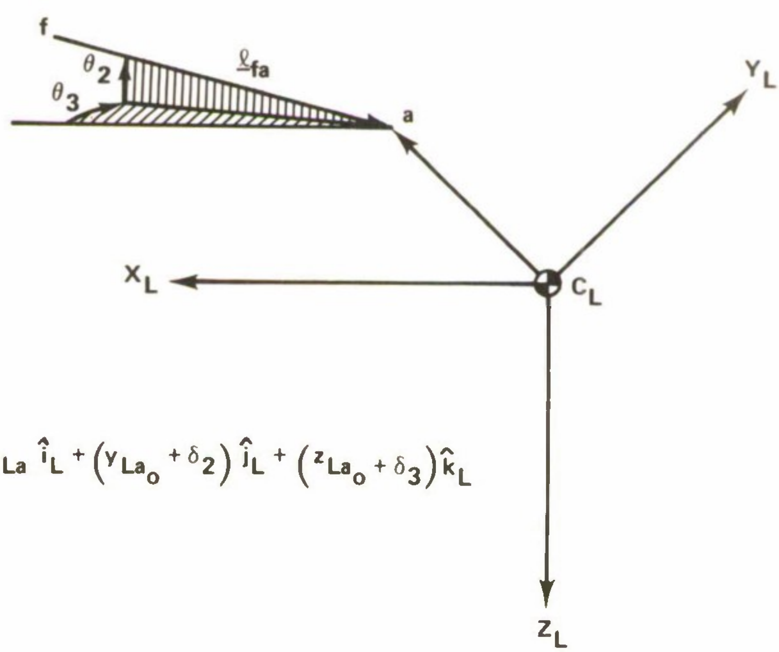

Figure 10. Displacements of points f and a.

where,

$$\underset{\sim}{\underline{\ell}}_{fa} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\ell_{fa} \\ 0 & \ell_{fa} & 0 \end{bmatrix} , \tag{71}$$

and ℓ_{fa} is a negative number, since it is the x_m-component of $\underline{\ell}_{fa}$ which lies on the rocket centerline and is directed from f to a.

(d) Frictional Force. Guidance mechanisms, such as rails and shoes, inherently are friction producers. The frictional force on the rocket opposes motion and in our model is assumed to act only parallel to the x_L-axis. Since the spring forces, $\underline{F}_{fs}$ and $\underline{F}_{as}$, act perpendicular to the x_L-axis, it is assumed that the frictional force's magnitude is $\mu(|\underline{F}_{as}| + |\underline{F}_{fs}|)$, where μ is a friction coefficient. The direction of the frictional force is assumed to be in the $-x_L$-component of the rocket's weight. Therefore,

$$\underline{F}_{mF} = -\mu\left(|\underline{F}_{fs}| + |\underline{F}_{as}|\right)\hat{i}_L \quad . \tag{72}$$

(e) Torque About C_m. The torque about C_m is due to the thrust, the springs, and friction. Explicitly, in vector form, we have for center-shoe-type rocket guidance,

$$\begin{aligned} \underline{T}_{m\,C_m} = &-(\underline{u} - \underline{\ell}_F) \times \underline{F}_T - (\underline{u} + \underline{\ell}_{fa}) \times \underline{F}_{fs} \\ &- \underline{u} \times \underline{F}_{as} - (-\underline{u} - \underline{\ell}_{fa} \pm \rho_f\, \hat{j}_L) \times \mu|\underline{F}_{fs}|\hat{i}_L \\ &- (-\underline{u} \pm \rho_a\, \hat{j}_L) \times \mu|\underline{F}_{as}|\, \hat{i}_L + (1 - \mu_s)\underline{T}_s \quad , \end{aligned} \tag{73}$$

where ρ_f and ρ_a are the distances to fore and aft rails, respectively, from the rocket centerline and the signs on ρ_f and ρ_a are determined by the signs of the y_L-components of $\underline{F}_{fs}$ and $\underline{F}_{as}$ so that the frictional forces act on either the right (+) or left (-) side of the rocket at the fore and aft shoe positions. Also, in Equation (73), $\underline{\ell}_F$, a vector from a to a point on the line of action $\underline{F}_T$ may be expressed as,

$$\underline{\ell}_F = X_F \hat{i}_m + \sigma_y \hat{j}_m + \sigma_z \hat{k} \quad , \tag{74}$$

where σ_y and σ_z are normally much less than the rocket's diameter and represent the amount of linear thrust malalignment. Finally, μ_s is the spin-torque friction coefficient and $\underline{T}_s$ is the spin torque.

A suitable matrix form for $\underline{T}_{m/C_m}$ (rocket basis) is

$$\begin{aligned} \underline{T}_{m/C_m} = &-(\tilde{\underline{u}} - \tilde{\underline{\ell}}_F)\, \underline{F}_T - (\tilde{\underline{u}} + \tilde{\underline{\ell}}_{fa})\, \underline{\underline{A}}\, \underline{F}_{fs} - \tilde{\underline{u}}\, \underline{\underline{A}}\, \underline{F}_{as} \\ &+ (\tilde{u} + \tilde{\underline{\ell}}_{fa} - \underline{\rho}_f)\, \mu |\underline{F}_{fs}| (100)^T \\ &+ (\tilde{\underline{u}} - \underline{\rho}_a)\, \mu |\underline{F}_{as}| (100)^T + (1 - \mu_s)\, \underline{T}_s \quad , \end{aligned} \tag{75}$$

where

$$\tilde{\underline{\rho}}_f = \pm \rho_f \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{76}$$

$$\tilde{\underline{\rho}}_a = \pm \rho_a \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad . \tag{77}$$

(f) Torque About a. The torque on the rocket about point a, $\underline{T}_{m/a}$, may be simply expressed in the matrix form,

$$\underline{T}_{m/a} = \underline{T}_{m/C_m} + \tilde{\underline{u}}\, \underline{\underline{A}}\, \underline{F}_m \quad . \tag{78}$$

(2) Torque on the System. The torque $\underline{T}_o$ is that about 0 due to external forces on the system of launcher plus rocket. We assume that the only external forces acting on the system are those due to gravity, the rocket's propulsion system and the launcher supporting structure which is modeled as pitch, yaw, and roll torsional springs.

It is assumed that the natural frequencies of the launcher's vibration in pitch, yaw, and roll are known. We designate these natural frequencies (undamped) by $\omega_{n_{y_L}}$, $\omega_{n_{z_L}}$, $\omega_{n_{x_L}}$, respectively, and the associated damping ratios by ζ_{y_L}, ζ_{z_L}, and ζ_{x_L}. We also assume that the launcher equations of motion can be written as

$$\underline{\underline{I}}_{L\,o}\,\underline{\ddot{\alpha}} + \underline{\underline{C}}_L\,\underline{\dot{\alpha}} + \underline{\underline{K}}_L\,\underline{\alpha} = \underline{f} \quad , \tag{79}$$

where $\underline{f}$ may be a nonlinear function of $\underline{\alpha} = (\alpha_1\ \alpha_2\ \alpha_3)^T$ and its derivatives and any other of the variables which define the system motion, and also as

$$\underline{\ddot{\alpha}} = 2\begin{bmatrix} \zeta_{x_L} & 0 & 0 \\ 0 & \zeta_{y_L} & 0 \\ 0 & 0 & \zeta_{z_L} \end{bmatrix}\begin{bmatrix} \omega_{n_{x_L}} & 0 & 0 \\ 0 & \omega_{n_{y_L}} & 0 \\ 0 & 0 & \omega_{n_{z_L}} \end{bmatrix}\underline{\dot{\alpha}}$$

$$+\begin{bmatrix} \omega^2_{n_{x_L}} & 0 & 0 \\ 0 & \omega^2_{n_{y_L}} & 0 \\ 0 & 0 & \omega^2_{n_{z_L}} \end{bmatrix}\underline{\alpha} = \underline{\underline{I}}^{-1}_{L/o}\,\underline{f} \quad , \tag{80}$$

so that,

$$\underline{\underline{C}}_L = \underline{\underline{I}}_{L/o}\begin{bmatrix} 2\zeta_{x_L}\,\omega_{n_{x_L}} & 0 & 0 \\ 0 & 2\zeta_{y_L}\,\omega_{n_{y_L}} & 0 \\ 0 & 0 & 2\zeta_{z_L}\,\omega_{n_{z_L}} \end{bmatrix} \tag{81}$$

and

$$\underline{\underline{K}}_L = \underline{\underline{I}}_{L/o} \begin{bmatrix} \omega^2_{n_{x_L}} & 0 & 0 \\ 0 & \omega^2_{n_{y_L}} & 0 \\ 0 & 0 & \omega^2_{n_{z_L}} \end{bmatrix} \tag{82}$$

are the launcher damping and stiffness matrices.

The torque about 0 may now be expressed in the matrix form (launcher basis),

$$\begin{aligned} \underline{T}_o = & -\underline{\underline{C}}_L \, \dot{\underline{\alpha}} - \underline{\underline{K}}_L \, \underline{\alpha} + \left(\tilde{\underline{\underline{R}}} + \tilde{\underline{S}}_a \, \underline{\underline{A}}^T \, \tilde{\underline{u}} \, \underline{\underline{A}}\right) \underline{\underline{B}} \, m\underline{g} + \tilde{\underline{\underline{R}}} \, \underline{\underline{B}} \, M\underline{g} \\ & + \left(\tilde{\underline{\underline{R}}} + \tilde{\underline{S}}_a + \underline{\underline{A}}^T \, \tilde{\underline{\ell}}_F \, \underline{\underline{A}}\right) \underline{\underline{A}}^T \, \underline{F}_T + \underline{\underline{A}}^T \, \underline{T}_s \quad . \end{aligned} \tag{83}$$

(3) Equilibrium Considerations. It is assumed that initially the system is in equilibrium under gravity and elastic structural forces. It is further assumed that the small elastic displacement of the rocket relative to the launcher may be neglected in determining the equilibrium orientation of the launcher, but that the equilibrium orientation of the launcher should be used in computing the rocket's relative equilibrium displacement.

(a) Launcher Equilibrium. Let $\underline{\alpha}_o$ denote the orientation vector of the launcher when the launcher springs are not deformed and let $\underline{\alpha}_e$ denote the change in $\underline{\alpha}$ which will result in a balance in the spring and gravitational forces. The equilibrium condition then is

$$m(\tilde{\underline{R}} + \tilde{\underline{S}}_a + \tilde{\underline{u}})\underline{\underline{B}}_e \, \underline{g} + M \, \tilde{\underline{R}} \, \underline{B}_e \, \underline{g} \doteq \underline{\underline{K}}_L \, \underline{\alpha}_e \quad , \tag{84}$$

where, without loss of generality, it is assumed that $\underline{\underline{A}} = \underline{\underline{E}}$ initially and where for small $|\underline{\alpha}_e|$

$$\underline{\underline{B}}_e \simeq -\tilde{\underline{\alpha}}_e \, \underline{\underline{B}}_o + \underline{\underline{B}}_o \quad , \tag{85}$$

with $\underline{\underline{B}}_o$ denoting the matrix $\underline{\underline{B}}$ in the undeformed state.

By using Equation (85) in Equation (84), we obtain

$$\left\{[m(\tilde{\underline{R}} + \tilde{\underline{S}}_a + \tilde{\underline{u}}) + M\tilde{\underline{R}}]\ (\widetilde{\underline{\underline{B}}_o \underline{g}}) - \underline{\underline{K}}_L\right\} \underline{\alpha}_e = \underline{Z} \quad , \tag{86}$$

where

$$\underline{Z} = -[m(\tilde{\underline{R}} + \tilde{\underline{S}}_a + \tilde{\underline{u}}) + M\tilde{\underline{R}}]\ \underline{\underline{B}}_o\ \underline{g} \quad . \tag{87}$$

Therefore,

$$\underline{\alpha}_e = -\left\{\underline{\underline{K}}_L - \left[m(\tilde{\underline{R}} + \tilde{\underline{S}}_a + \tilde{\underline{u}}) - M\tilde{\underline{R}}\right] (\widetilde{\underline{\underline{B}}_o\ \underline{g}})\right\}^{-1} \underline{Z}_1 \tag{88}$$

and the initial value of $\underline{\alpha} = \underline{\alpha}_o + \underline{\alpha}_e$.

(b) Rocket Equilibrium. Since the rocket may translate in the y_L- and z_L-directions (x_L — motion constrained) as well as rotate, two vector equilibrium conditions must be satisfied; i.e., the sum of the forces acting on the rocket must be zero and the sum of the torques about a point on the rocket, say point a, must also vanish.

Summing forces, we have

$$\underline{F}_{fs} + \underline{F}_{as} + m\ \underline{\underline{B}}_e\ \underline{g} = \underline{0} \tag{89}$$

and summing torques about a, we find that

$$-\tilde{\underline{\ell}}_{fa}\ \underline{F}_{fs} + m\ \tilde{\underline{u}}\ \underline{\underline{B}}_e\ \underline{g} = \underline{0} \quad . \tag{90}$$

It follows, from Equations (89) and (90), that

$$\tilde{\underline{\ell}}_{fa}\ \underline{F}_{as} = -m(\tilde{\underline{u}} + \tilde{\underline{\ell}}_{fa})\underline{\underline{B}}_e\ \underline{g} \quad . \tag{91}$$

Let

$$mg\underline{b} = m(\tilde{\underline{u}} + \tilde{\underline{\ell}}_{fa})\underline{\underline{B}}_e\ \underline{g} \quad . \tag{92}$$

Then, the equilibrium values of δ_2 and δ_3 are

$$\delta_{2_e} = \frac{-mgb_2}{x_{fa}\ k_{a_{33}}} \tag{93}$$

and

$$\delta_{3_e} = \frac{mgb_3}{x_{fa}\, k_{a_{22}}} \quad , \tag{94}$$

where $x_{fa}\hat{i}_m = \underline{\ell}_{fa}$ and $k_{a_{22}}$ and $k_{a_{33}}$ are the elements of $\underline{\underline{K}}_a$ indicated by the subscripts.

Once δ_{2_e} and δ_{3_e} are known, θ_{2_e} and θ_{3_e} may be obtained from

$$\theta_{2_e} = \frac{-c_2}{x_{fa}^2\, k_{f_{33}}} \tag{95}$$

and

$$\theta_{3_e} = \frac{-c_3}{x_{fa}^2\, k_{f_{22}}} \quad , \tag{96}$$

where c_2 and c_3 are the indicated elements of

$$\underline{c} = m\, \underline{\tilde{\ell}}_{fa}\, \underline{\underline{B}}_e\, \underline{g} - \underline{\tilde{\ell}}_{fa}\, \underline{\underline{K}}_f \quad . \tag{97}$$

g. Rocket Inertia Matrix

The centroidal inertia matrix of the rocket, $\underline{\underline{I}}_m$, is obtained by assuming that the principal centroidal inertia matrix, $\underline{\underline{I}}_{p_m}$, is known. Let

$$\underline{\underline{I}}_{p_m} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \quad , \tag{98}$$

and let

$$\underline{\underline{D}} \cong \begin{bmatrix} 1 & \mu_3 & -\mu_2 \\ -\mu_3 & 1 & 0 \\ \mu_2 & 0 & 1 \end{bmatrix} \tag{99}$$

denote the matrix required to transform from the $C_m x_m y_m z_m$ system to the principal system $C_m x_p y_p z_p$ (Figure 11). Then, $\underline{\underline{I}}_m$ is given by

$$\underline{\underline{I}}_m = \underline{\underline{D}}^T \, \underline{\underline{I}}_{p_m} \, \underline{\underline{D}} \tag{100}$$

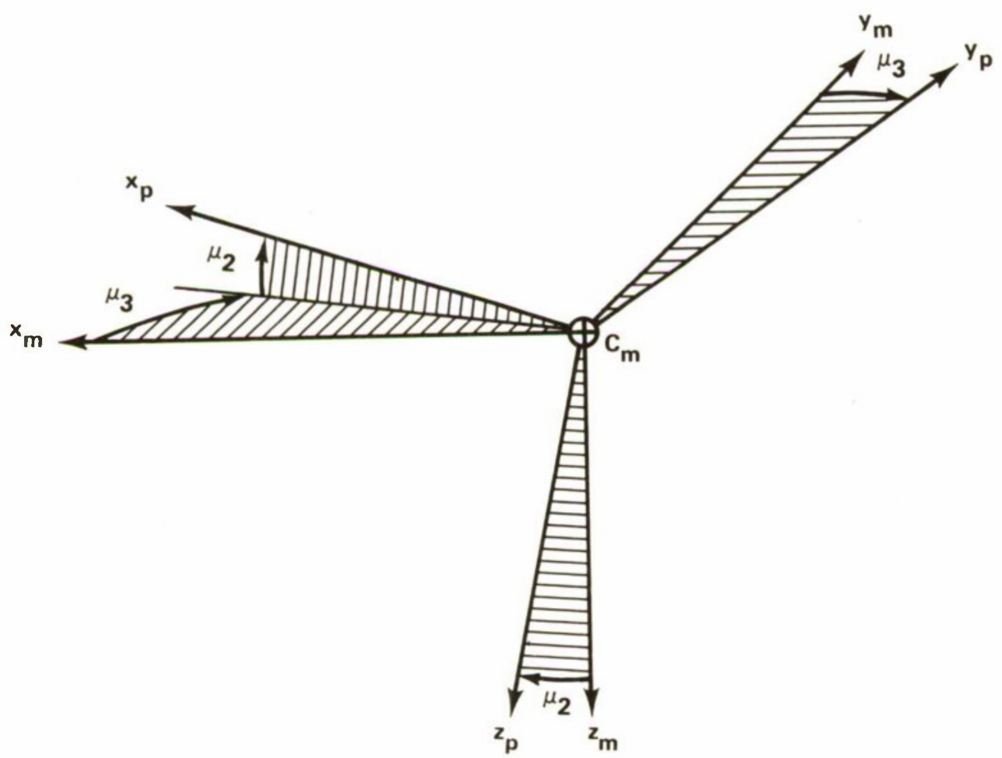

Figure 11. Rocket-fixed and principal coordinate systems.

which, neglecting squares and products of μ_2 and μ_3, explicitly is

$$\underline{\underline{I}}_m \simeq \begin{bmatrix} I_1 & \mu_3(I_1 - I_2) & -\mu_2(I_1 - I_3) \\ \mu_3(I_1 - I_2) & I_2 & 0 \\ -\mu_2(I_1 - I_3) & 0 & I_3 \end{bmatrix} . \tag{101}$$

Since we may express $\underline{\underline{I}}_m$ as

$$\underline{\underline{I}}_m = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} , \tag{102}$$

it follows from Equations (101) and (102) that

$$I_{xx} \simeq I_1 , \quad I_{xy} \simeq -\mu_3(I_2 - I_1) ,$$

$$I_{yy} \simeq I_2 , \quad I_{xz} \simeq \mu_2(I_3 - I_1) ,$$

$$I_{zz} \simeq I_3 \text{ and } I_{yz} \simeq 0 .$$

Section IV. SIMULATION METHODS

a. General Comments

The equations which govern the motion of the system have been completely defined except for the magnitudes of the spin torque and thrust. Since the manner in which the time variations of these quantities is generated is more of simulation topic, it is therefore treated in this section. This section also discusses matrix manipulations used in generating the right-hand sides of the time derivatives of the state variables and the method of integration used to solve the set of 27 differential equations.

b. Generation of Spin Torque and Thrust

The magnitudes of the spin torque and the thrust are generated for computational purposes in the same manner; hence, we will describe the procedure for the thrust magnitude, F.

Values of F at times when derivatives are needed are obtained by assuming that the thrust profile is piece-wise linear. In the present version of the simulation program, four values of the thrust magnitude, F_j, j = 1,2,3,4, at times t_o, j = 1,2,3,4, are used for this purpose (Figure 12). When $T_j \leq t < t_{j+1}$, the value of F is found from

$$F = F_j + (F_{j+1} - F_j)(t - t_j)(t_{j+1} - t_j) \quad . \tag{103}$$

For $t < t_1$ (in the computer code, $T5 \equiv t_1$, $T6 \equiv t_2$, ... $T9 = t_5$), $F = 0$ and for $t > t_5$, $F = 0$.

c. Matrix Algebra

The matrix operations which are needed to generate the derivatives used in the integration process are (1) multiplication of a 3×3 matrix times a 3×1 matrix, (2) multiplication of two 3×3 matrices, (3) generation of a "tilde" matrix from a 3×1 matrix, (4) inversion of a 3×3 matrix, and (5) matrix addition, subtraction and transposition. Matrix subtraction, addition, and transposition are carried out directly when necessary during the computations by using DO loops. The other operations, or manipulations, are carried out in separate subroutines, all of which are very straightforward because all elements of the result are calculated explicitly even in the matrix inversion. For completeness, the algorithms of the subroutines MATXV (3×3 times 3×1), MATMPY (3×3 times 3×3), TILDE (form 3×3 matrix from 3×1), and MATINV (inverse of 3×3) are given as follows:†

†These operations are, of course, well known.

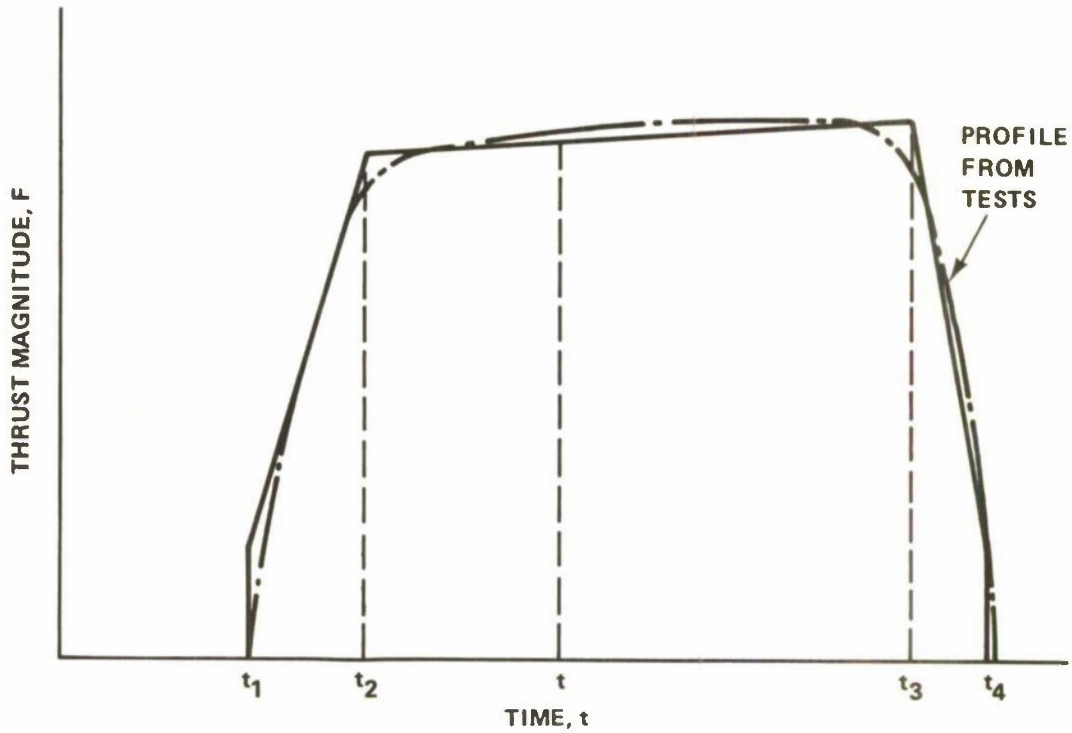

Figure 12. Thrust magnitude versus time.

MATXV

Input:

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad . \tag{104}$$

Operations:

$$w_j = \sum_k a_{jk} v_k, \; k = 1,2,3, \; j = 1,2,3 \quad . \tag{105}$$

Output:

$$\underline{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad . \tag{106}$$

MATMPY

Input:

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{107}$$

$$\underline{\underline{B}} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad . \tag{108}$$

Operations:

$$c_{ij} = \sum_{k=1}^{3} a_{ik}\, b_{kj} \quad . \tag{109}$$

Output:

$$\underline{\underline{D}} = \underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \quad . \tag{110}$$

TILDE

Input:

$$\underline{X} = \left(x_1\ x_2\ x_3\right)^T \quad . \tag{111}$$

Output:

$$\underline{\tilde{X}} = \begin{bmatrix} 0 & -X_3 & X_2 \\ X_3 & 0 & -X_1 \\ -X_2 & X_1 & 0 \end{bmatrix} \quad . \tag{112}$$

MATINV

Input:

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad . \tag{113}$$

Operations:

$$\begin{aligned} D = {} & a_{11} (a_{22} a_{33} - a_{32} a_{33}) \\ & + a_{12} (a_{31} a_{23} - a_{21} a_{33}) \\ & + a_{13} (a_{21} a_{32} - a_{31} a_{22}) \quad . \end{aligned} \tag{114}$$

$$\begin{aligned} b_{11} &= (a_{22} a_{33} - a_{32} a_{23})/D \\ b_{12} &= (a_{32} a_{13} - a_{12} a_{33})/D \\ b_{13} &= (a_{12} a_{23} - a_{22} a_{13})/D \\ b_{21} &= (a_{31} a_{23} - a_{21} a_{33})/D \\ b_{22} &= (a_{11} a_{33} - a_{31} a_{13})/D \\ b_{23} &= (a_{21} a_{13} - a_{11} a_{23})/D \\ b_{31} &= (a_{21} a_{32} - a_{22} a_{31})/D \\ b_{32} &= (a_{31} a_{12} - a_{32} a_{11})/D \\ b_{33} &= (a_{11} a_{22} - a_{12} a_{21})/D \quad . \end{aligned} \tag{115}$$

Output:

$$\underline{\underline{B}} \ , \quad \mathrm{DET} = D \quad .$$

Inversion of the 6×6 matrix $\underline{\underline{J}}$ has already been discussed in Subsection III.f.

d. Integration Method

Various methods may be used to solve sets of first-order ordinary differential equations. The most popular type of self-starting algorithm is, however, the Runge-Kutta, and a fourth-order algorithm of this type was chosen for use in this study.

The fourth-order algorithm is basically as follows [7]:

Given the state vector $\underline{x}$ (a 27 × 1 matrix in this case) at time t and the time derivative functions f_j (x,t); i.e.,

$$\dot{\underline{x}} = \underline{f}(x,t) \quad , \tag{116}$$

1) Compute:

$$\underline{w}_1 = f(\underline{x}(t),t)\Delta t \quad . \tag{117}$$

where Δt is the step size.

2) Compute:

$$\underline{u}_1 = \underline{x}(t) + \underline{w}_1/2 \quad . \tag{118}$$

3) Compute:

$$\underline{w}_2 = f(\underline{u}_1,t + \Delta t/2)\Delta t \tag{119}$$

4) Compute:

$$\underline{u}_2 = \underline{x}(t) + \underline{w}_2/2 \quad . \tag{120}$$

5) Compute:

$$\underline{w}_3 = \underline{f}(\underline{u}_2,t + \Delta t/2)\Delta t \quad . \tag{121}$$

6) Compute:

$$\underline{u}_3 = x(t) + \underline{w}_3 \quad . \tag{122}$$

7) Compute:

$$\underline{w}_4 = \underline{f}(\underline{u}_3,t + \Delta t)\Delta t \quad . \tag{123}$$

8) Compute the new state vector, $\underline{x}(t + \Delta t)$, from

$$\underline{x}(t + \Delta t) = \underline{x}(t) + (\underline{w}_1 + 2\underline{w}_2 + 2\underline{w}_3 + \underline{w}_4)/6 \quad . \tag{124}$$

By redefining t to be t + Δt and $\underline{x}(t)$ to be $\underline{x}(t + \Delta t)$, one may repeat steps 1) through 8) until t is as large as desired, or until one of the elements of $\underline{x}$ has attained a certain value.

Section V. SIMULATION RESULTS

a. <u>General Comments</u>

As a check on the theory and the computer, several test runs were made using data which, it is emphasized, do not represent any actual launcher/rocket system, but some of which does correspond to physical data for the AERO Wind Sensitivity Rocket currently awaiting tests by the US Army Missile Command [8]. The data used to obtain the results presented here are given in the appendix as an example of the input needed for the computer program.

Only a few of the more interesting plots which were obtained during this study are presented in the next subsection.

b. <u>Discussion of Some Typical Results</u>

Plots of various variables are shown in Figures 13 through 28. The plots were made and are exhibited to illustrate the capabilities of the program and also some of the effects of the mallaunch factors: (1) detent force, (2) mass unbalance, and (3) thrust malalignment.

Five simulation runs were made. These were obtained in the following manner:

1) Run 1 – No detent force, mass unbalance, or thrust malalignment were included.

2) Run 2 – A detent force of 1800 pounds (maximum magnitude) was applied.

3) Run 3 – The detent force of Run 2 was retained and a mass unbalance caused by rotating the principal axis of the rocket through an angle $\mu_2 = 0.00001^*$ radian (about the y_m-axis) was included.

4) Run 4 – The detent force was retained and a thrust malalignment due to $\alpha_y = 0.0001^*$ radian was included.

5) Run 5 – The detent force, mass unbalance of Run 3, and thrust malalignment of Run 4 were all included.

Figures 13, 14, and 15 show the time histories of the launcher roll, pitch, and yaw angles, respectively. Since the launcher is relatively stiff and massive (compared to other possible launchers), the disturbing influences of mass unbalance and thrust malalignment on the launcher motion are negligible compared to the effects of the detent force.

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*This is relatively small compared to values which may be expected.

Note that holding the rocket during spin-up caused a noticeable variation in launcher roll and yaw. However, noticeable is probably a misnomer if angles of less than 10^{-3} radians are considered negligible. The detent force has a noticeable effect on roll, pitch, and yaw, being sufficient to cause a pitch angle difference of about 0.00005 radian (amplitude).

Figures 16 and 17 show the body-fixed (missile-fixed) angular velocity components Ω_2 and Ω_3 as functions of time. The thrust comes on when $\phi \simeq 3.66$ radians and the missile leaves the launcher when $\phi \simeq 13.25$ radians, and abrupt changes in the slopes of Ω_2 and Ω_3 are noticeable at these values of ϕ. The choice of mass unbalance and thrust malalignment happened to be such that they tend to cancel each other until the thrust cuts off when $\phi \simeq 28$ radians. Even after thrust cutoff, the mass unbalance effects persist, but for Run 5 the oscillations in Ω_2 and Ω_3 are of smaller amplitude than their Run 3 (mass unbalance only) counterparts.

The variables Ω_2 and Ω_3 are plotted in Figure 18. This figure is a little cluttered, but the almost identical traces for the last four runs until the rocket leaves the constraint of the launcher can be seen. After leaving the launcher in Run 3, a circle* centered at (-0.00105, 0) is described, indicating a bias in angular rate Ω_3 of -0.00105 rad/sec. During Run 4, smaller circles located approximately at (0.0008, 0) and (0, 0) are described. The combined effects of mass unbalance and thrust unbalance result in the curve labeled 5 which ultimately (after thrust cutoff) produces the small circle with center at (-0.00105, 0).

Figures 19 and 20 illustrate the behavior of the missile pitch rate and yaw rate for the five cases. The oscillations in $\dot{\theta}$ and $\dot{\psi}$ for Runs 3 and 4 are clearly 180° out of phase (as they should be) and for Run 5, the effects of mass unbalance and thrust malalignment almost cancel out until thrust cutoff.

Pitch rate is plotted versus yaw rate in Figure 21. The curves shown are similar to those in Figure 18, but the ellipses described after launch by the curves for Runs 3, 4, and 5 are observed to wobble; i.e., the curves do not close as those of Figure 18. This is due to long-period changes in $\dot{\theta}$ and $\dot{\psi}$ resulting from the small difference in the spin rate and the rate of precession. This difference divided by the spin rate is approximately $I_{x_m x_m}/I_{y_m y_m}$ (assuming $I_{y_m y_m} \cong I_{z_m z_m}$).

*Note the scale difference in Ω_2 and Ω_3 which result in circles rather than ellipses on Figure 18.

Curves showing rocket pitch angle versus rocket yaw angle are presented in Figure 22. The deviation due to thrust malalignment is much less than that due to mass unbalance for the values of μ_2 and α_y used. The coupling of mass unbalance and thrust malalignment is evident in the curve for Run 5. The change in yaw angle is much greater than that in θ for the parameters used as is evidenced by Figures 22, 23, and 24.

A combination of variables often used in deciding on the "goodness" of a launch is the square root of the sum of the squares of $\Delta\theta$ and ψ. This combination is shown as a function of time in Figure 25. It is merely fortuitous that the thrust malalignment detent combination resulted in the smallest value of $\sqrt{(\Delta\theta)^2 + \psi^2}$ at the end of the simulation time, since both mass unbalance and thrust malalignment are normally random quantities.

The variable θ_2 is given as a function of time in Figure 26. Since θ_2 is the pitch angle of the rocket with respect to the launcher, the time history of θ_2 differs in all cases (Runs) from that for $\Delta\theta$; i.e., θ_2 includes the launcher pitching motion ($\theta_2 \simeq \theta - \alpha_2$).

Figure 27 illustrates the climb rate and lateral velocity of the rocket's center of mass. Since the initial elevation angle of the rocket $\alpha_{20} + \alpha_{2e} \simeq 0.093422$ radian, the rocket's center of mass will ascend (this implies the Z becomes more negative), although gravity and other factors will slow its ascent slightly.

The last figure (Figure 28) illustrates the fact that the rocket's center of mass drops below the initial aim axis due to launcher's pitching down, gravity, and the other effects included in the various runs.

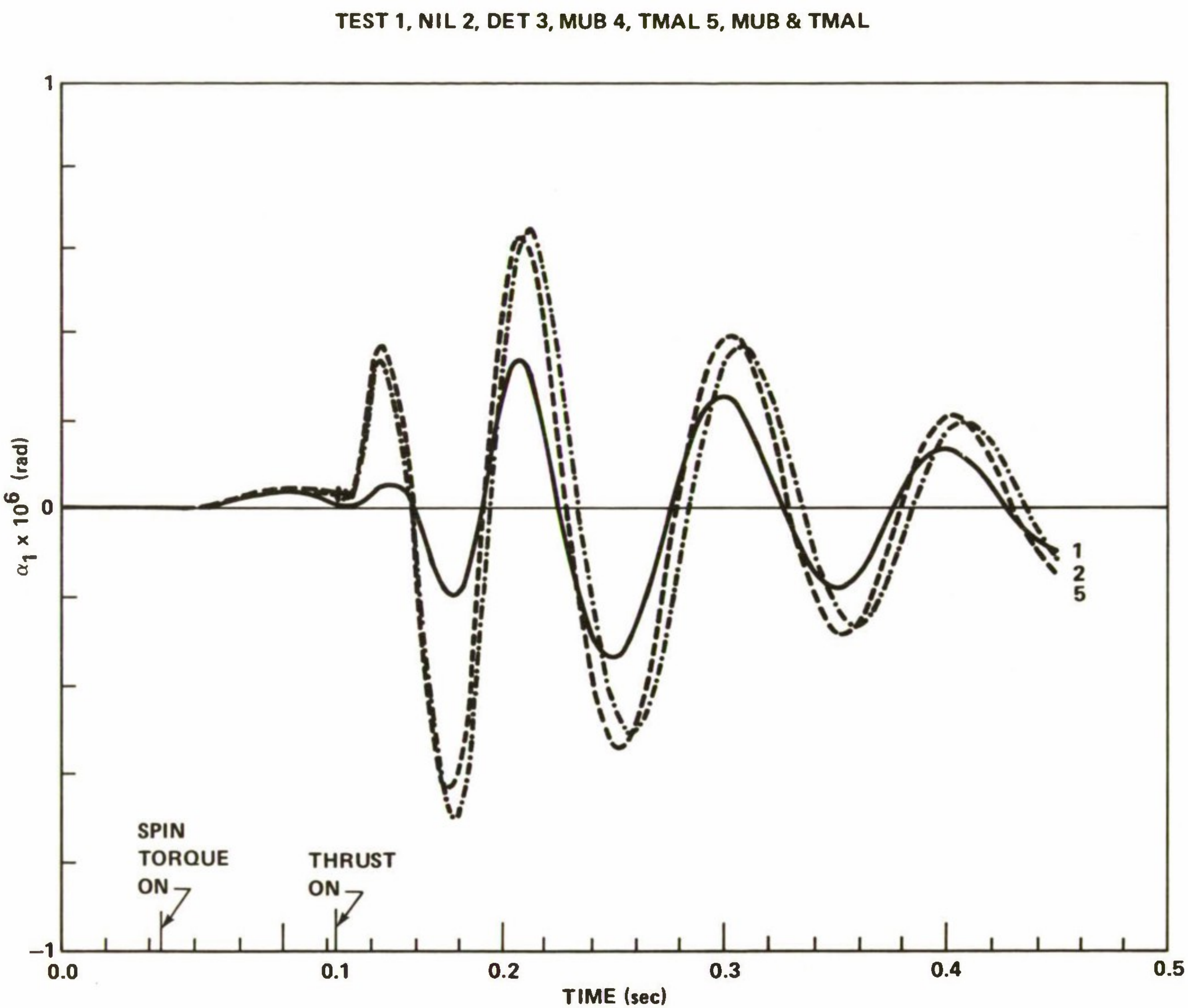

Figure 13. Launcher roll angle.

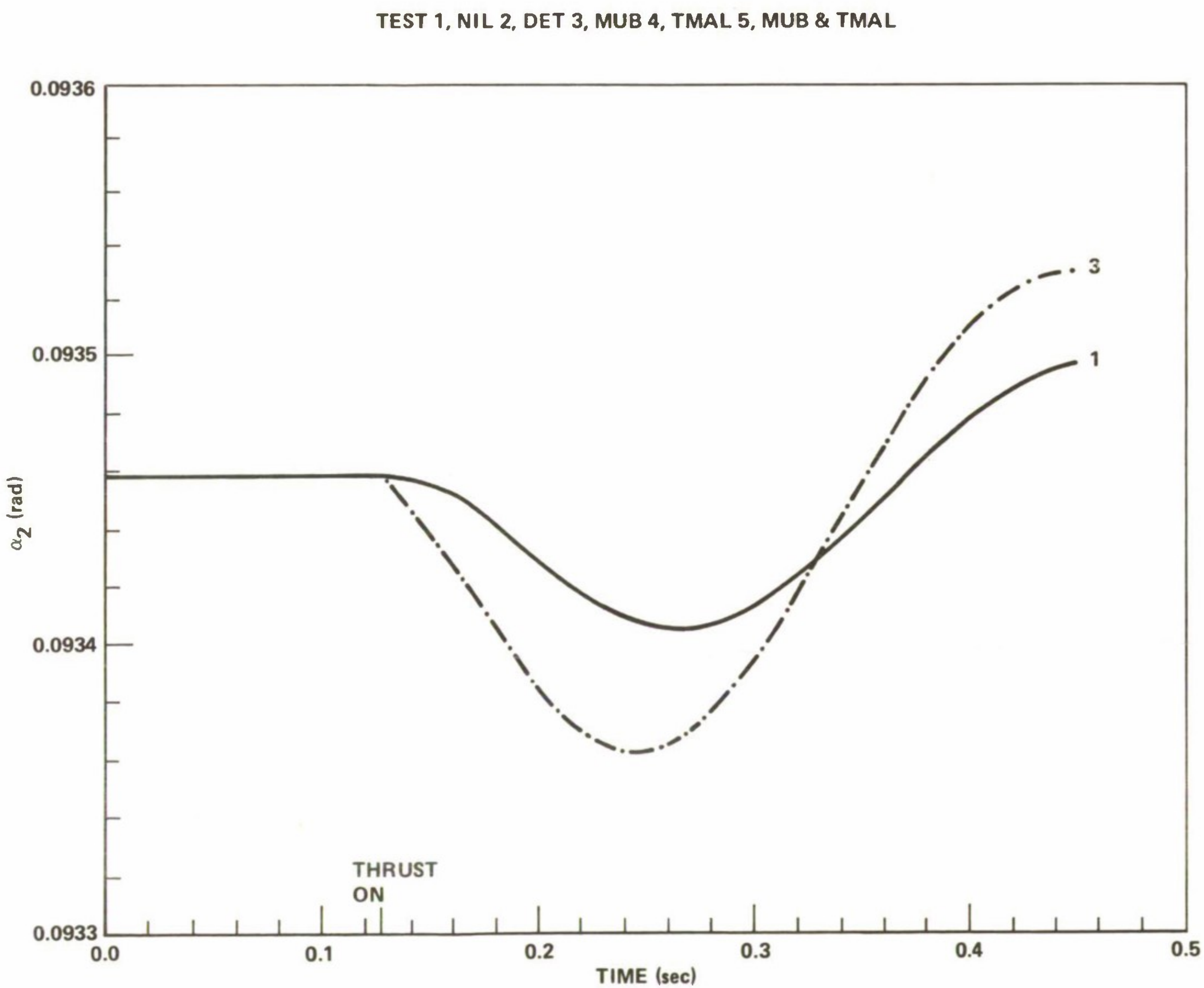

Figure 14. Launcher pitch angle.

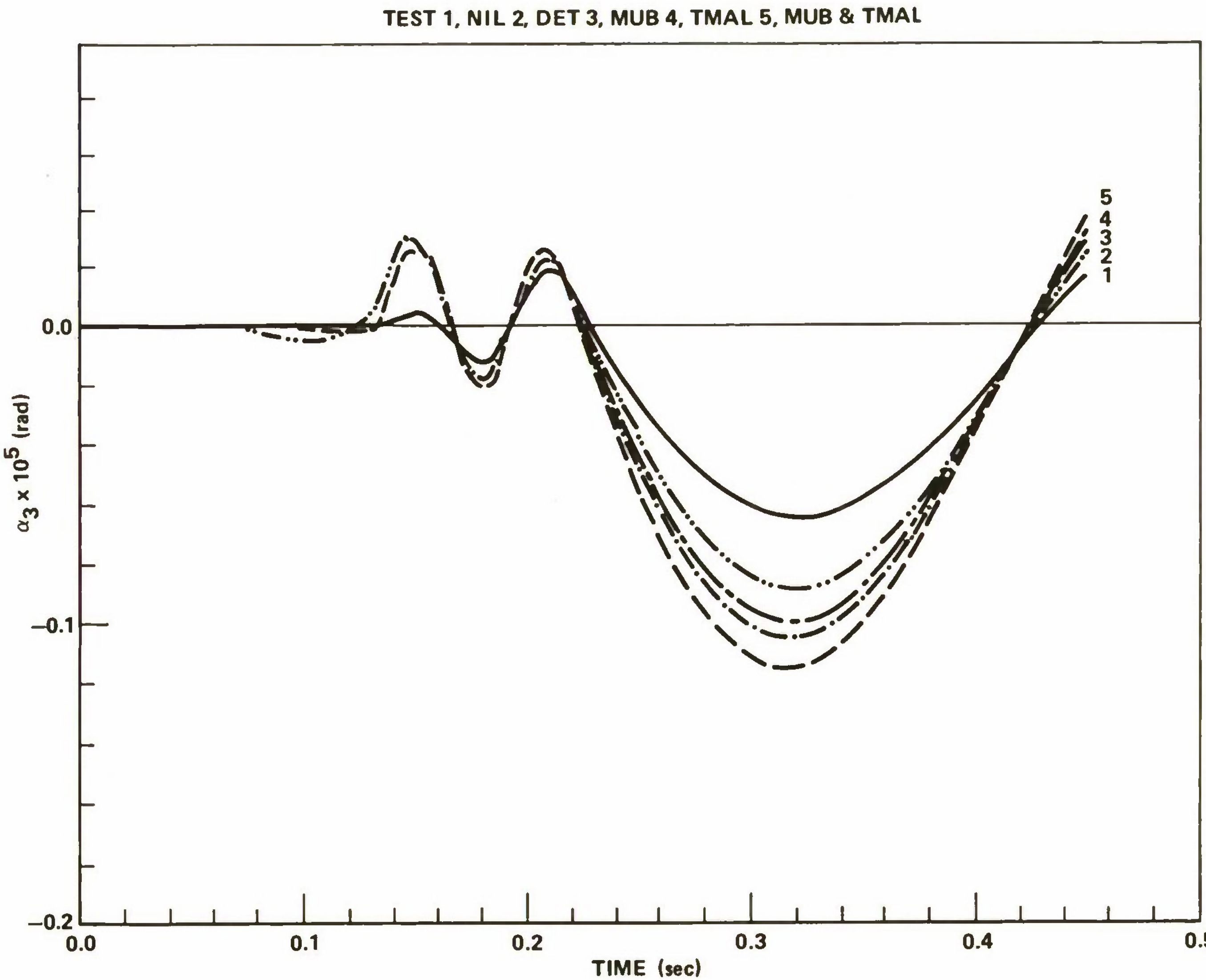

Figure 15. Launcher yaw angle.

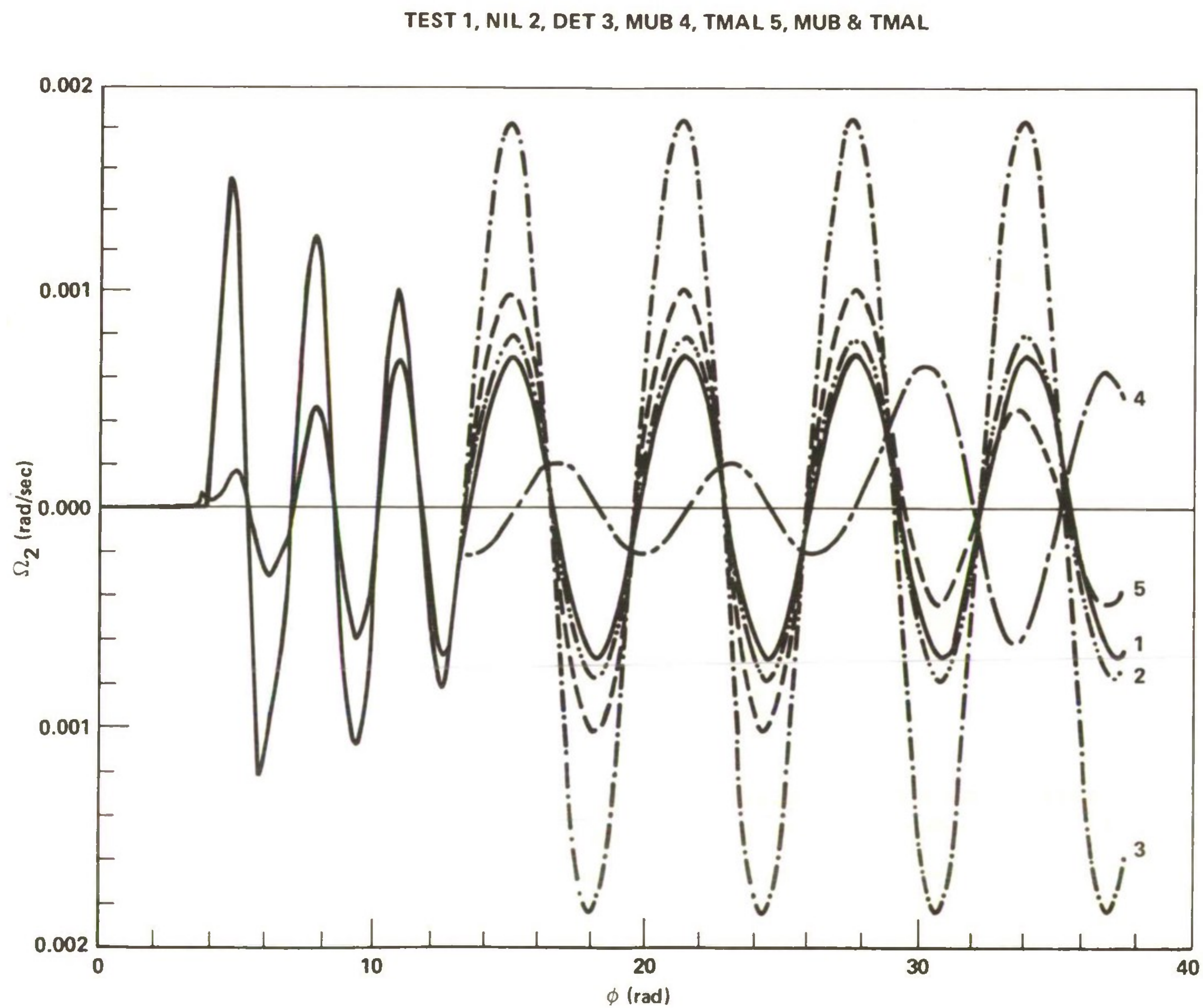

Figure 16. Body-fixed angular velocity component, Ω_2 versus roll angle, ϕ.

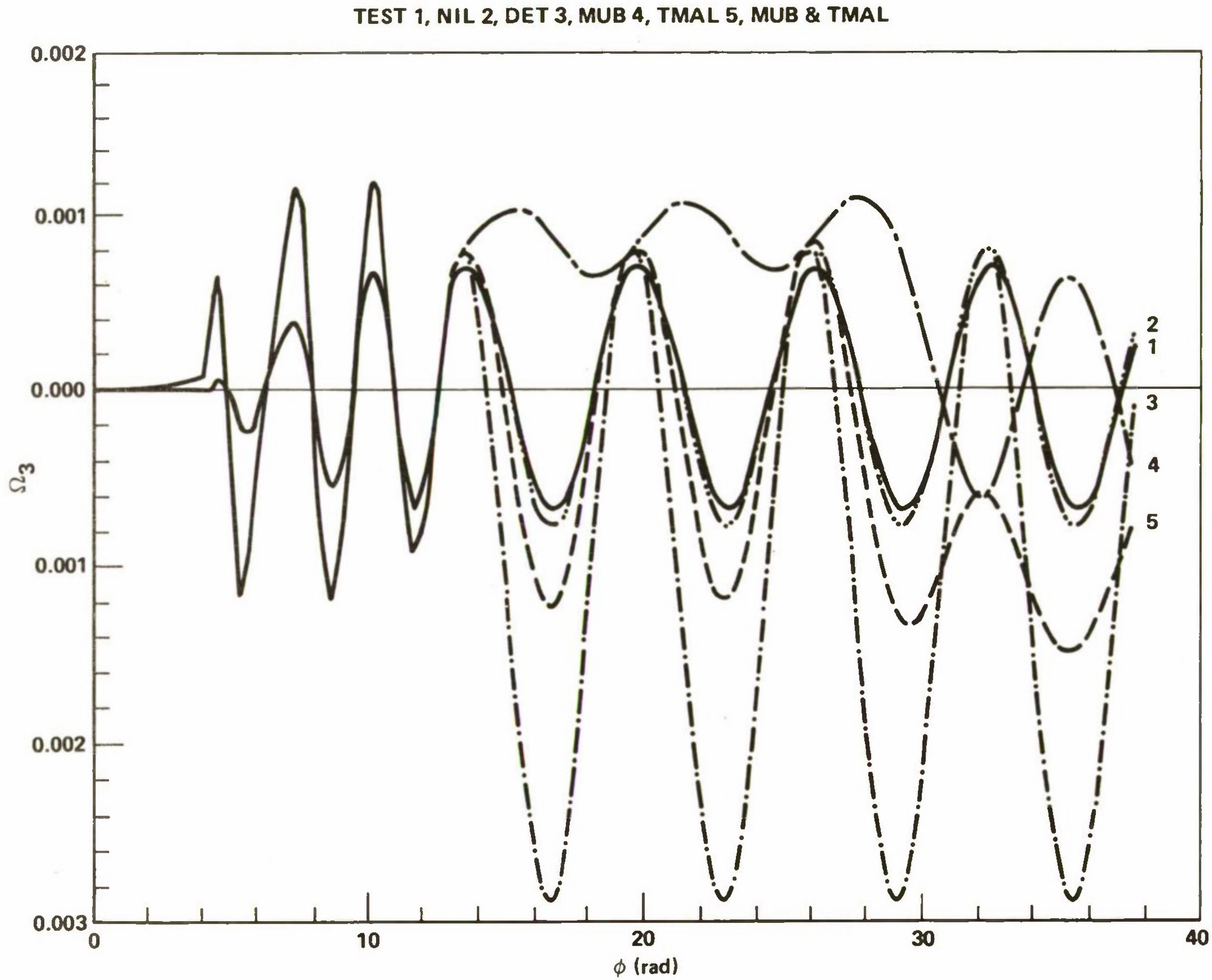

Figure 17. Body-fixed angular velocity component, Ω_3 versus roll angle ϕ.

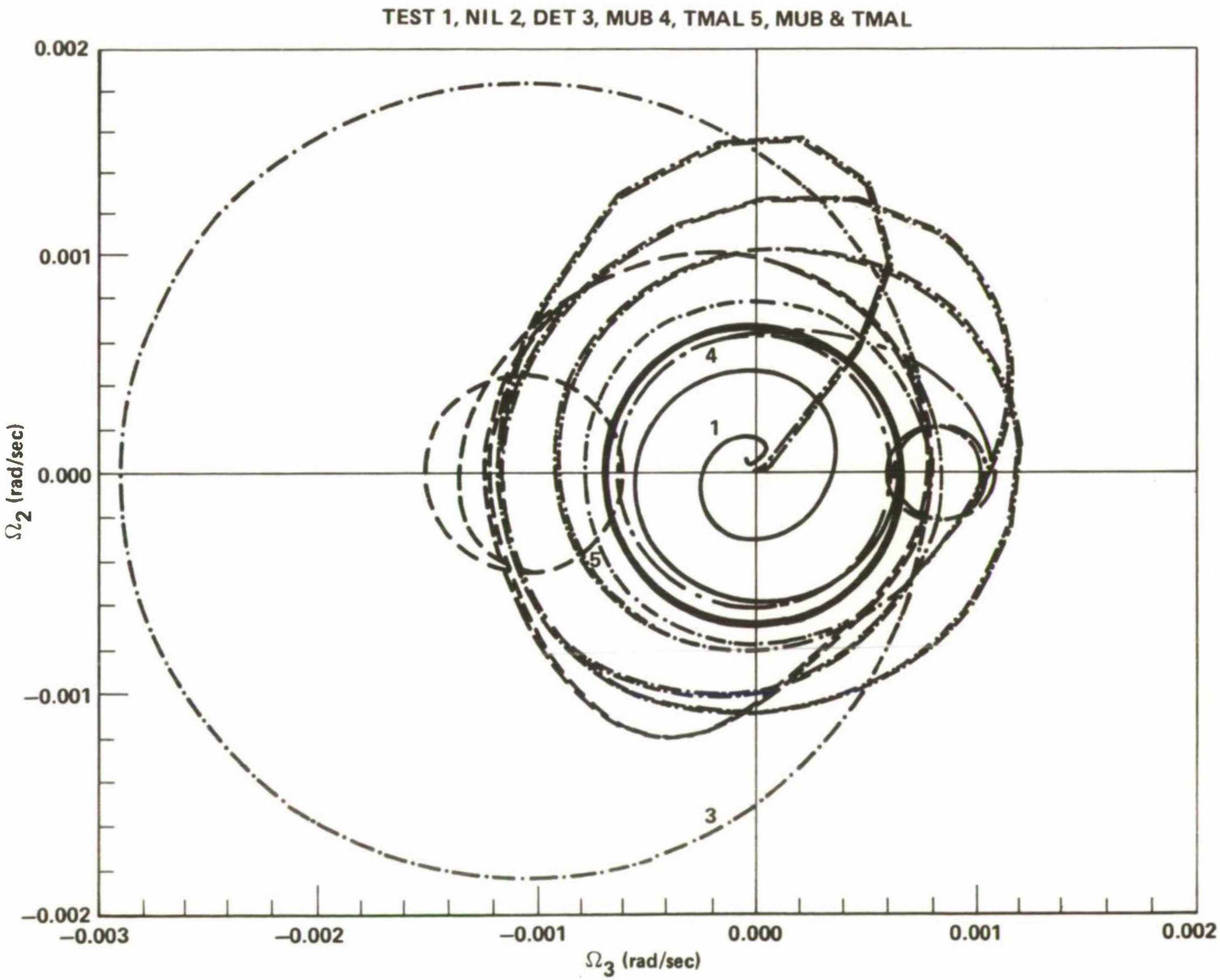

Figure 18. Body-fixed angular velocity components Ω_2 and Ω_3.

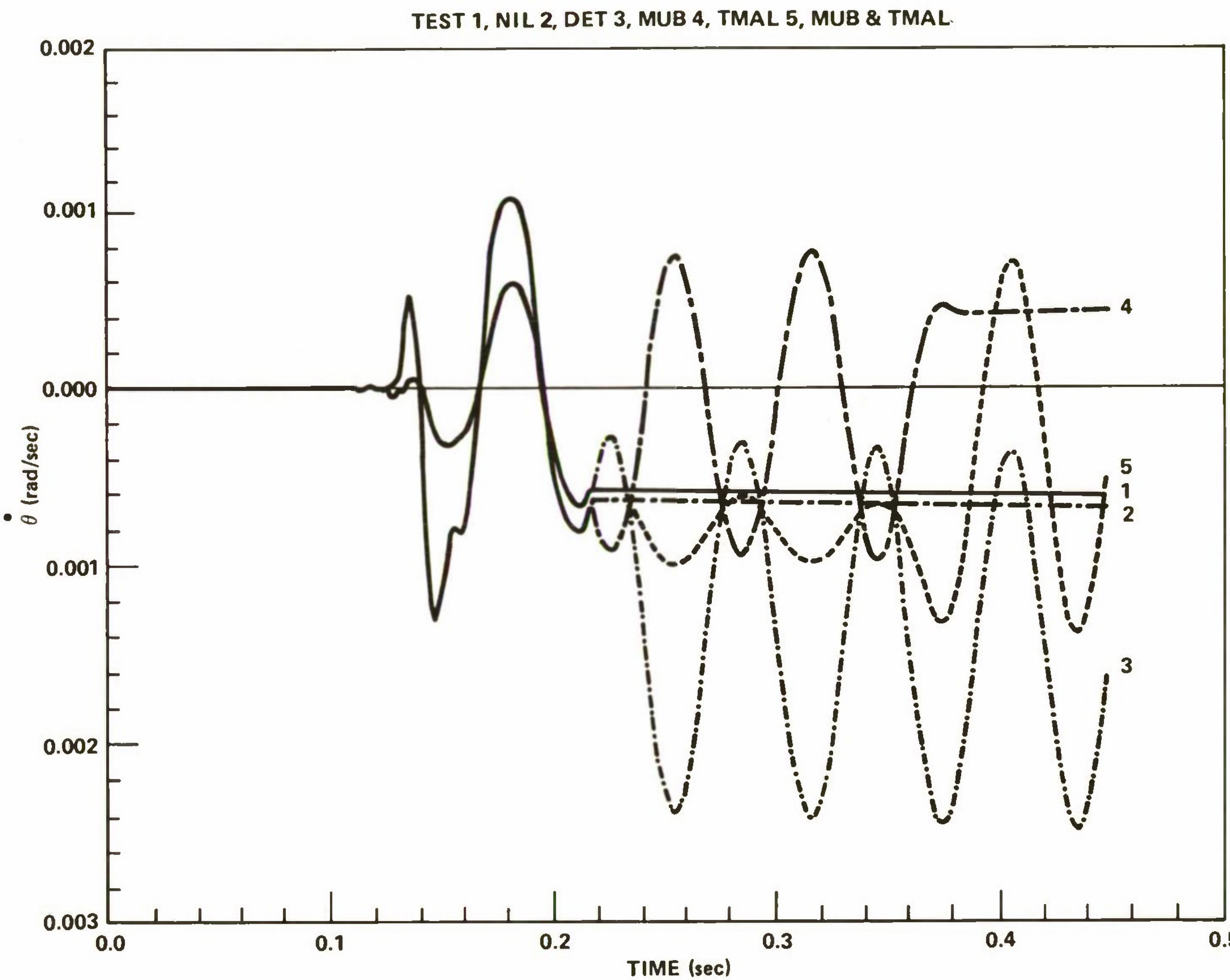

Figure 19. Pitch rate versus time.

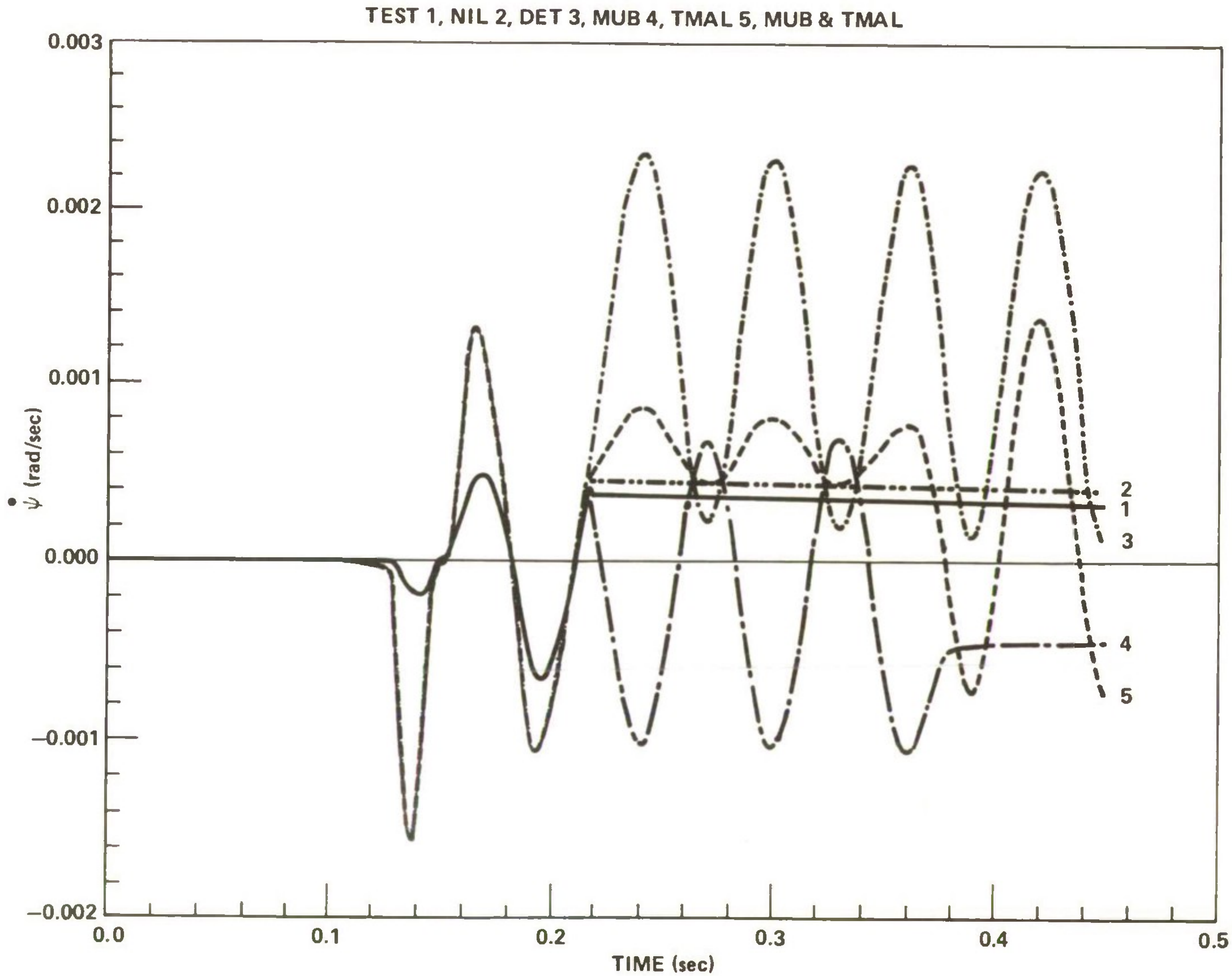

Figure 20. Yaw rate versus time.

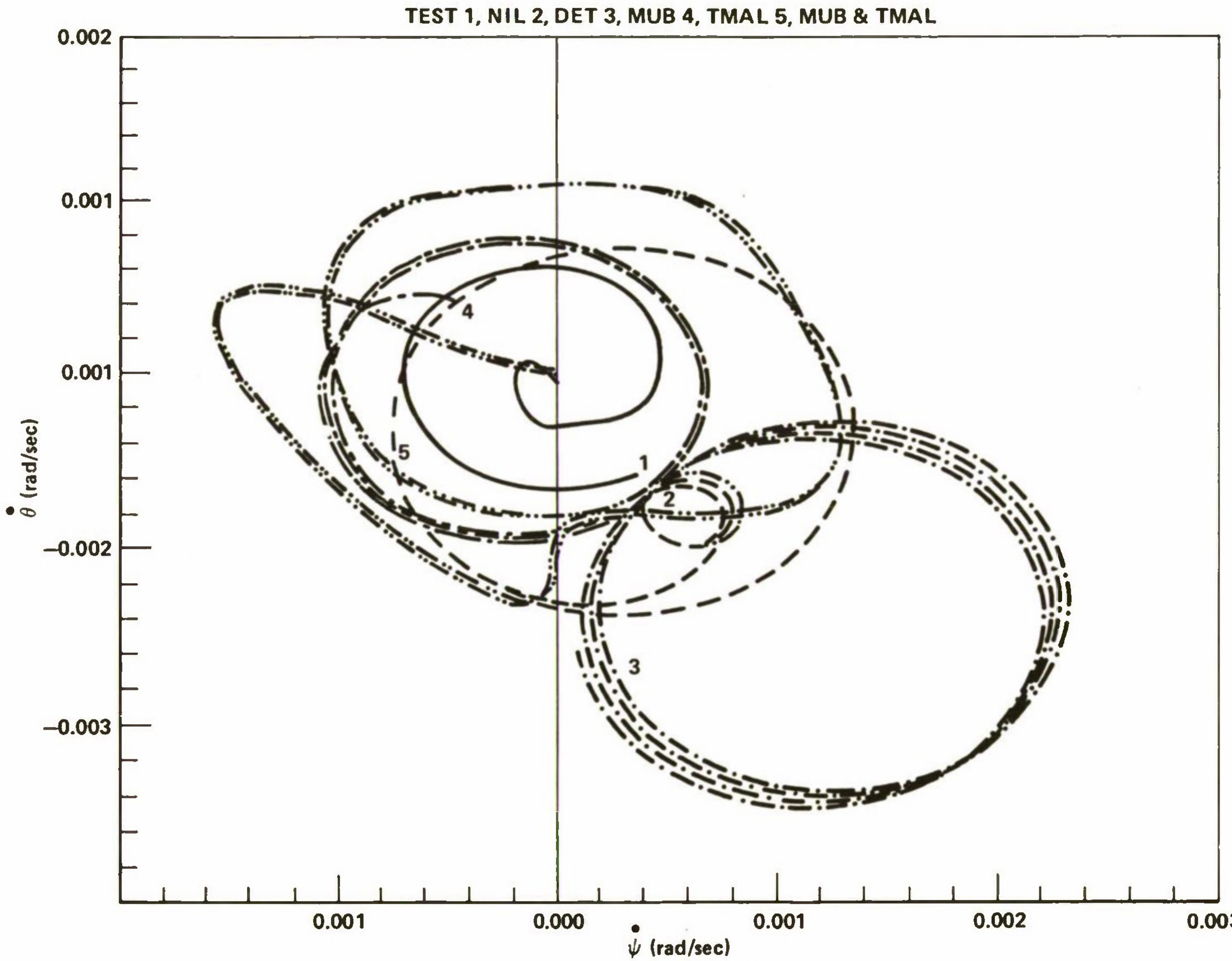

Figure 21. Pitch rate versus yaw rate.

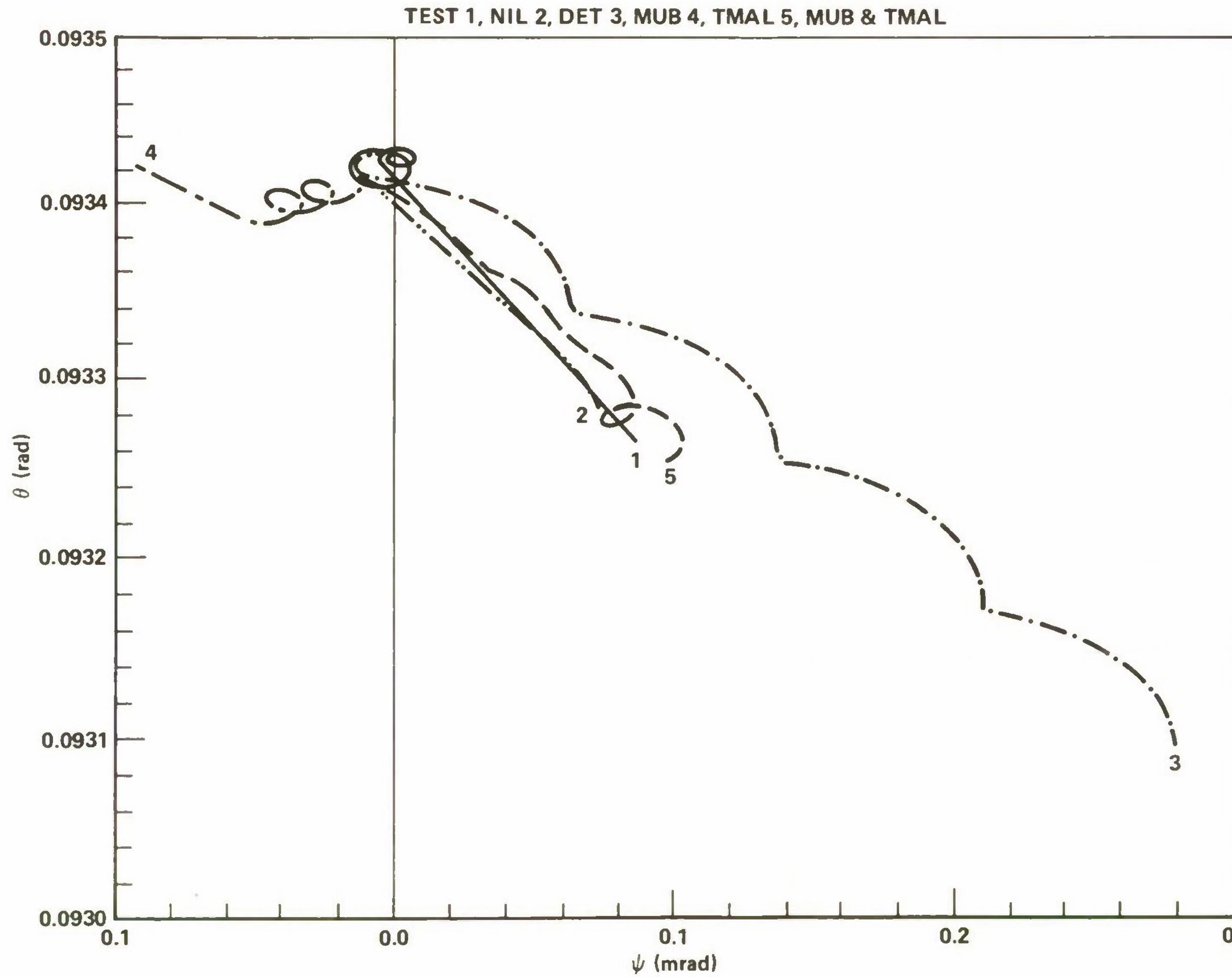

Figure 22. Pitch angle versus yaw angle.

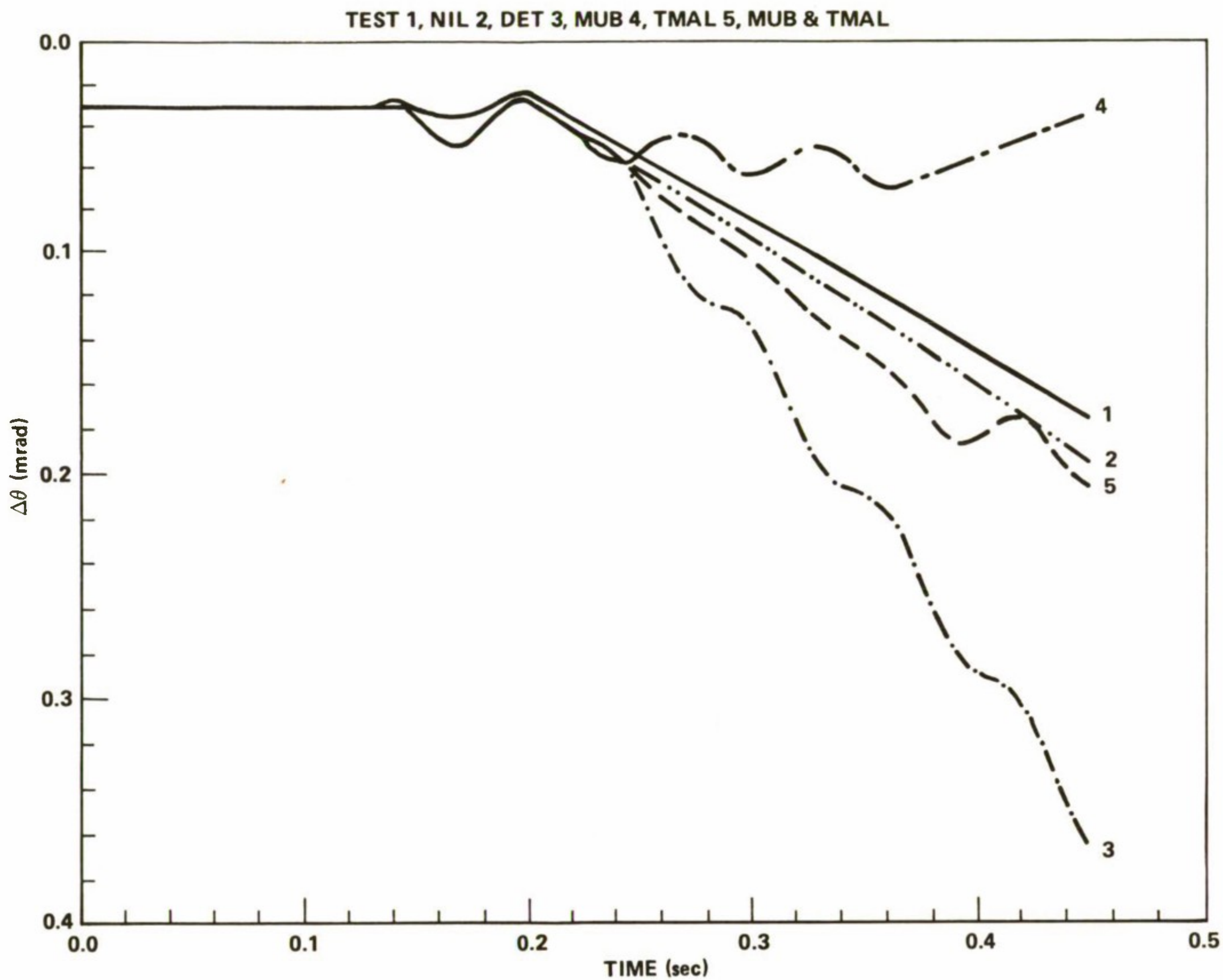

Figure 23. Rocket pitch angle with respect to aim axis versus time.

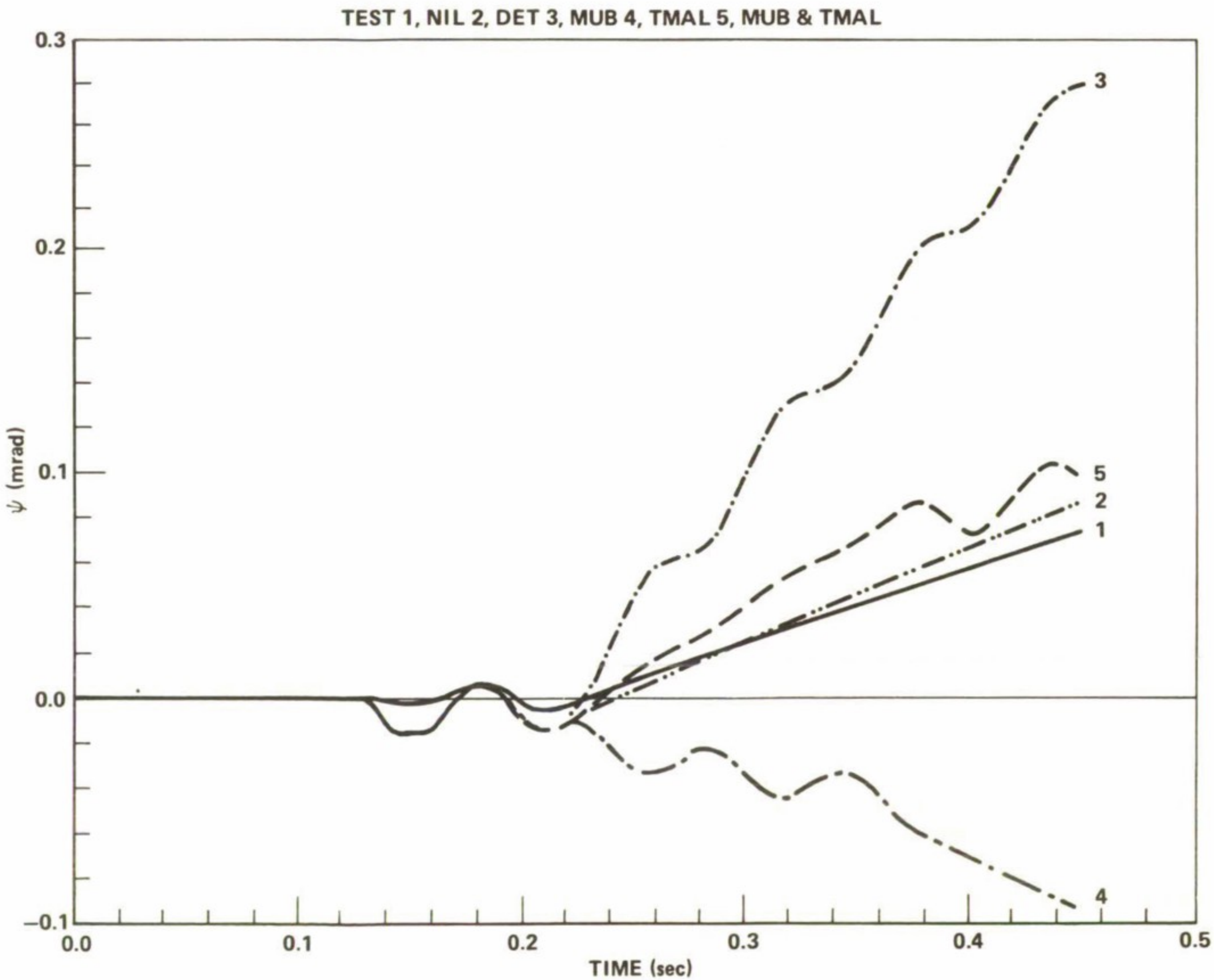

Figure 24. Rocket yaw angle versus time.

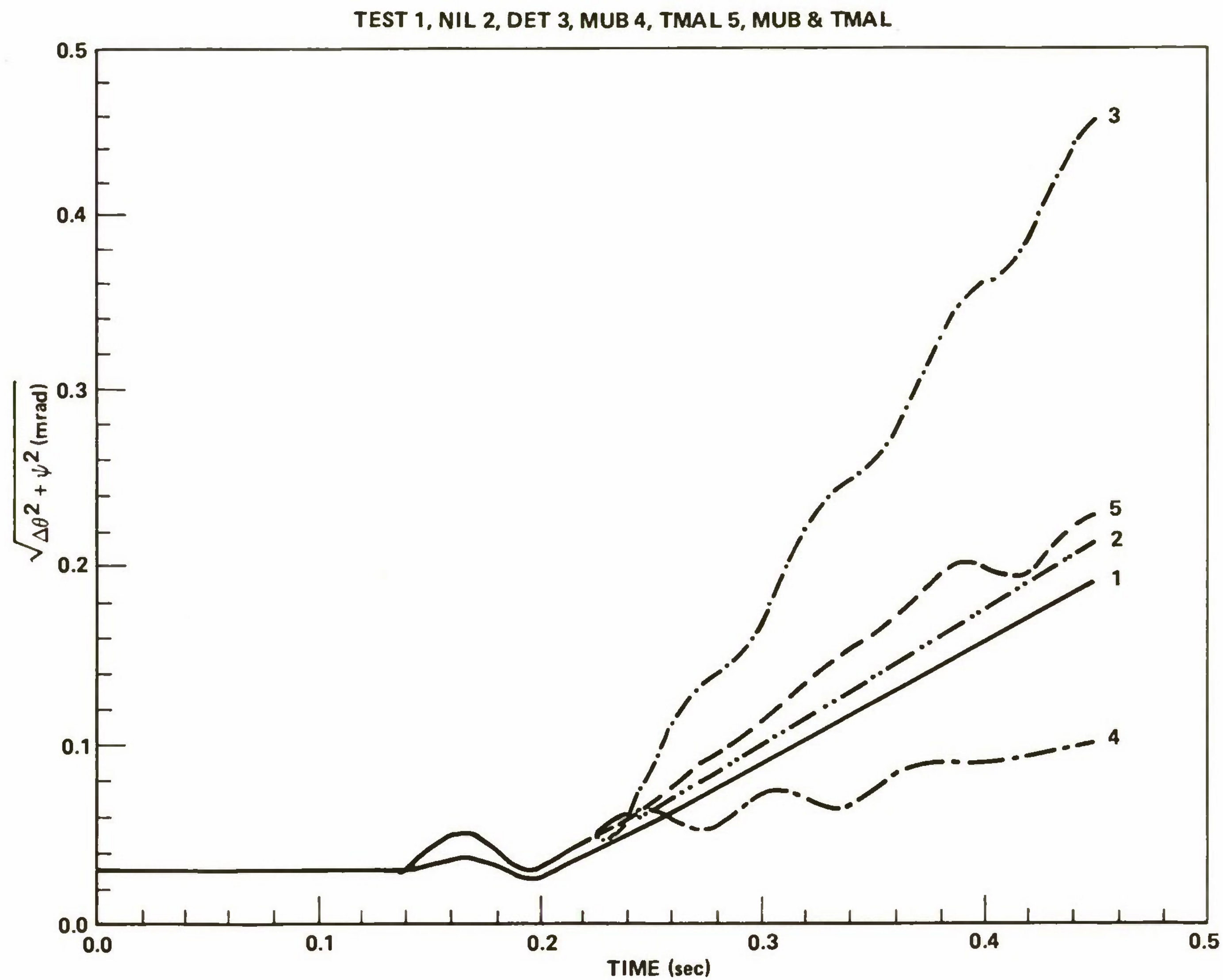

Figure 25. Square root of the sum of the squares of $\Delta\theta$ and ψ versus time.

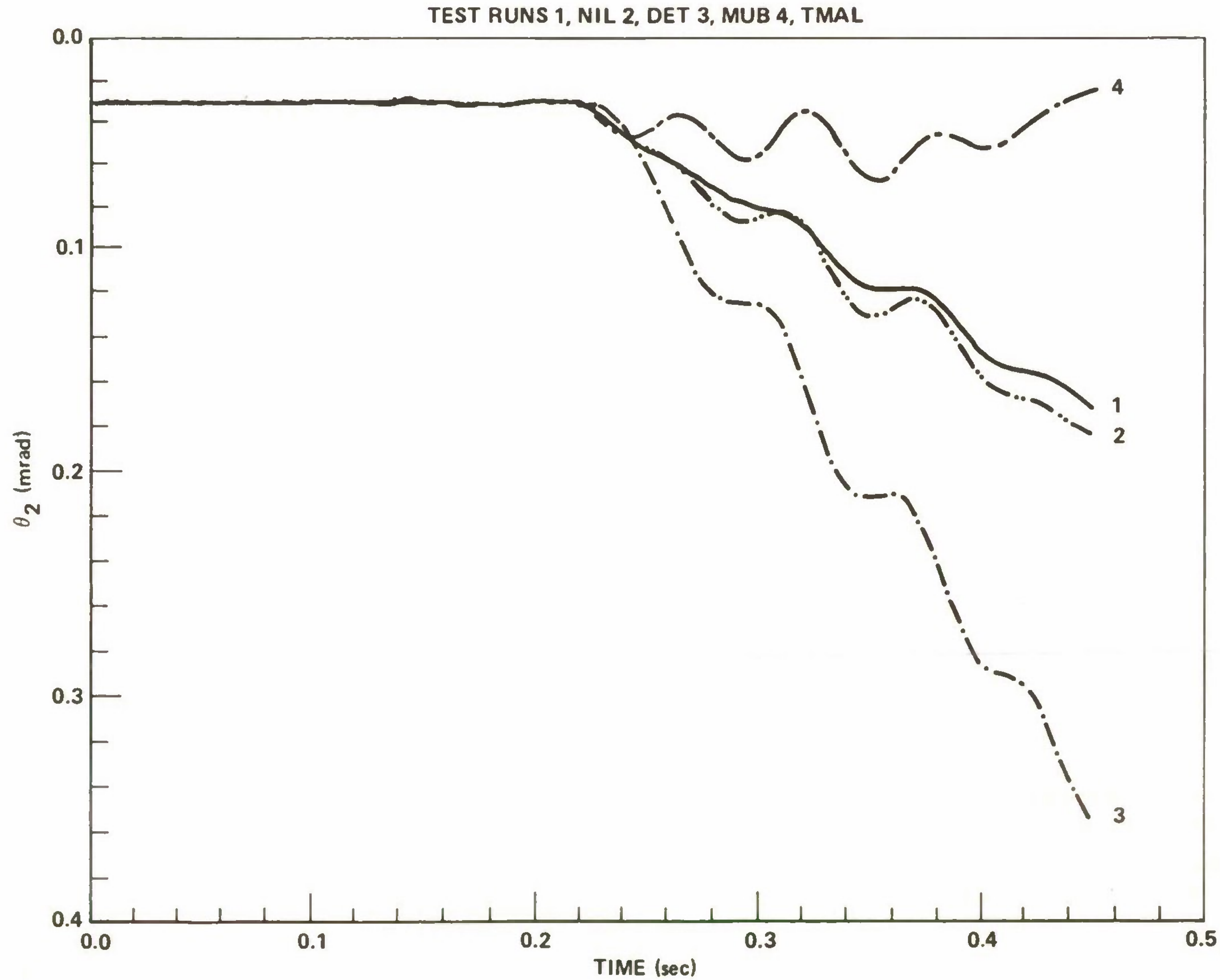

Figure 26. Pitch angle of the rocket with respect to the launcher versus time.

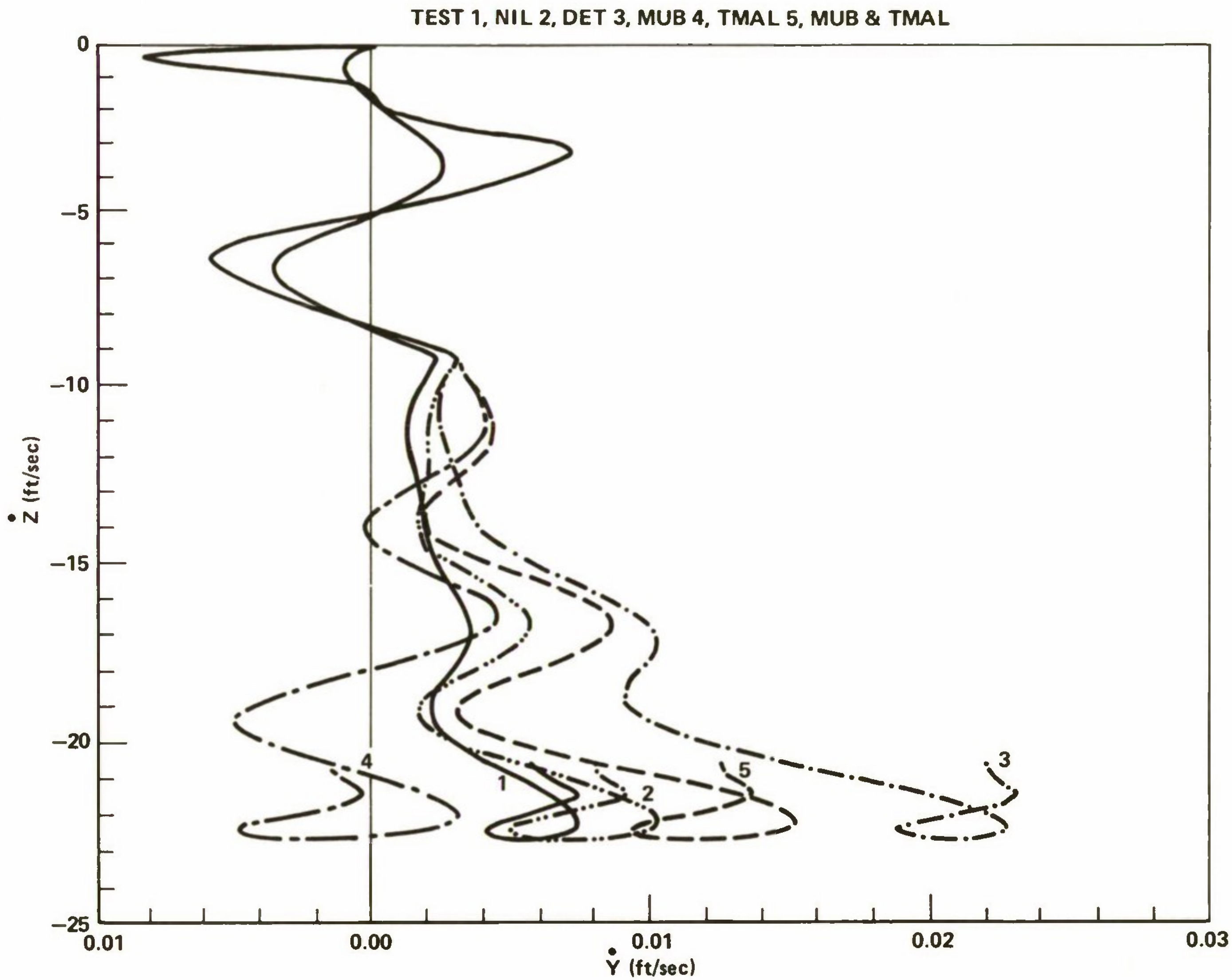

Figure 27. Inertial components of the velocity of the rocket's center of mass, $\dot{Z}$ versus $\dot{Y}$.

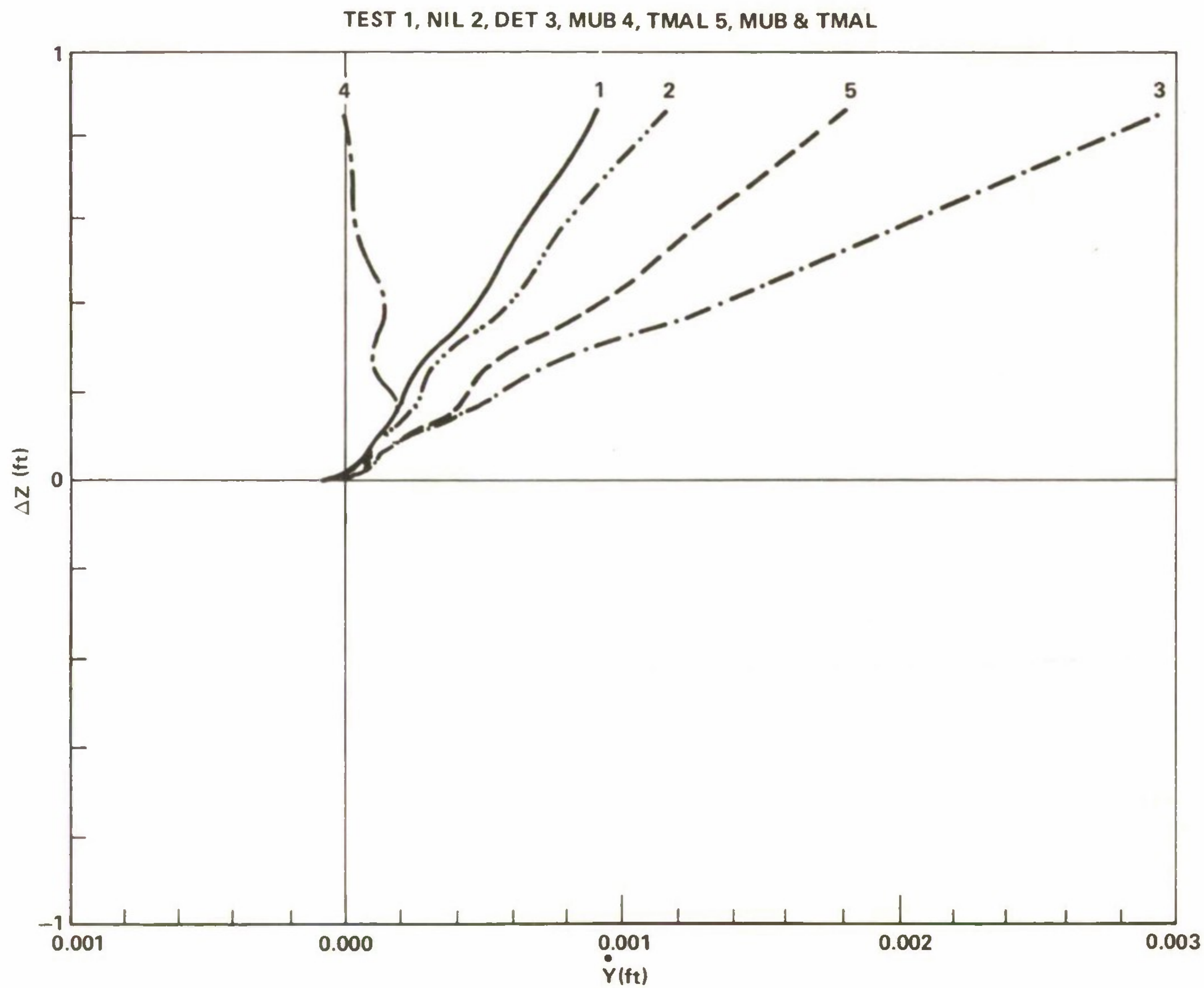

Figure 28. Position of rocket's center of mass with respect to the aim axis.

Section VI. SUMMARY AND CONCLUSIONS

a. Summary

The problem of simulating the dynamical behavior of a launcher/rocket system during the entire launch, i.e., from a state of rest with the rocket on the launch, through the detent, guidance, tipoff, and free-flight phases, has been considered. A physical model of the system which includes various factors that contribute to mallaunch of free rockets has been described. Dynamical and kinematical equations developed on the basis of the physical model have been presented and, where necessary, converted from vector to matrix form. The matrix equations have been incorporated in a digital computer code for numerical solution. The computer code has been used to produce preliminary results and these have been presented and discussed.

b. Conclusions

At the time of the writing of this report, the computer code had not been operational for a sufficient period of time to provide the data required to reach any major conclusions regarding mallaunch. The code should be useful, however, in future investigations.

Some conclusions which are already essentially generally known are:

1) Dynamic mass unbalance is a primary contributor to mallaunch of spinning free rockets.

2) Angular thrust malalignment is also an important mallaunch factor.

3) Dynamic mass unbalance and angular thrust malalignment effects may either tend to cancel or to compliment each other.

The preliminary results obtained indicate that the computer code has been properly implemented in its present form. It should, however, be modified to include the effects of aerodynamic forces and moments and of time varying rocket mass characteristics during the free-flight phase. Such work is planned under an ARD Grant to the author.*

For plans of future work in this area, the reader may contact the author or Dr. James L. Batson, US Army Missile Command, ATTN: AMSMI-RLH, Redstone Arsenal, Alabama 35805.

*A post-LRCP grant has been awarded to the author. The study funded by this grant will be conducted at Auburn University, Auburn, Alabama 36830.

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Appendix

INPUT/OUTPUT DESCRIPTION

CONTROL CARDS AND SAMPLE INPUT

```
OMVLT.
ACCT (PN=COCHRAN, PBC=7294L10000,CC=7800,JN=0001,OP=A2)
REQUEST(TAPE20,*PF)
REWIND(TAPE20)
ATTACH(FILE,LHDYN,ID=LHJEC,CY=2)
REWIND(FILE)
FILE.
REWIND(TAPE20)
CATALOG(TAPE20,LHXXXJC,ID=LHXXX,CY=5)
EXIT.
0000000000000000000000

DEMONSTRATION RUN THRUST MAL AND MASS UNBAL (ALY=0.0001, MU2=0.0001)
FINAL TIME DELT MIN DELT IMPRINT IPLOT
0.4500      0.0001      0.0001      5000     15
                          MISSILE PRINCIPAL INERTIA MATRIX
   0.29                      0.00                     0.00
  0.00                      70.8                       0.00
   0.00                     0.00                      70.8
  MU2      MU3
0.00001     0.0
                          LAUNCHER INERTIA MATRIX
 4800.0                     0.0                     1600.0
  0.0                       16000.0                 0.0
 1600.0                     0.0                     15200.0
                        FORE SHOE       STIFFNESS MATRIX
  0.00                      0.00                     0.00
 0.0                         1000000.0               0.0
 0.0                        0.0                     1000000.0
                        AFT SHOE STIFFNESS MATRIX
   0.00                      0.00                        0.00
0.0                          2000000.0              0.0
0.0                              0.0                2000000.0
                        LAUNCHER NATURAL FREQUENCIES
  10.0                      2.5                     2.5
                        LAUNCHER DAMPING RATIOS
 0.1                       0.1                      0.1
                        FORE SHOE DAMPING MATRIX
 0.00                      0.00                      0.0
0.0                         600.0                   0.0
0.0                         0.0                     600.0
                        AFT SHOE DAMPING MATRIX
  0.00                      0.00                    0.00
0.0                         800.0                   0.0
0.0                           0.0                   800.0
```

```
 M                SM  RHOF   RHOA      ALY          ALZ      SIGMAY   SIGMAZ
200.0            7.76 0.25   0.25    0.0001         0.00      0.0       0.0
 XLA              YLA  ZLA   MU      SPINT 1        SPINT 2  SPINT3   SPINT 4
-3.75             0.0 -2.0   0.1     0.0            450.0    450.0    450.0
T1                T2   T3    T4      T5             T6       T7       T8
0.05            0.065 0.125   0.1251 0.126           0.146    0.356   0.386
T9                F1   F2    F3      F4
 0.386          0.0   9350.0 10500.0 0.0
DETENT           SMU
1800.0           0.001
                      U
5.75                  0.0               0.0
                      G
0.0                     0.0                32.175
                      R
4.0                   0.0            -2.0
                      LF
-0.5                  0.0            0.0
                      LFA
-6.583                0.0               0.0
                      ALPHAO
0.0                   0.10           0.0
XLA1            XLA2
 0.25           0.25
ISHOE
 2
0000000000000000000000
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INPUT FORMATS

Card No.	Format	Card No.	Format
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2	3F10.0	24	8F10.0
3	3F25.0	25	8F10.0
4	3F25.0	26	8F10.0
5	3F25.0	27	8F10.0
6	2F10.0	28	8F10.0
7	3F25.0	29	3F20.0
8	3F25.0	30	3F20.0
9	3F25.0	31	3F20.0
10	3F25.0	32	3F20.0
11	3F25.0	33	3F20.0
12	3F25.0	34	3F20.0
13	3F25.0	35	2F10.0
14	3F25.0	36	I2
15	3F25.0		
16	3F25.0		
17	3F25.0		
18	3F25.0		
19	3F25.0		
20	3F25.0		
21	3F25.0		
22	3F25.0		

CONTROL CARDS AND SAMPLE INPUT

```
OMVLT.
ACCT(PN=COCHRAN,PBC=7294L10000,CC=7800,JN=0001,OP=A2)
REQUEST(TAPE20,*PF)
REWIND(TAPE20)
ATTACH(FILE,LHDYN,ID=LHJEC,CY=2)
REWIND(FILE)
FILE.
REWIND(TAPE20)
CATALOG(TAPE20,LHXXXJC,ID=LHXXX,CY=5)
EXIT.
0000000000000000000000
 DEMONSTRATION RUN THRUST MAL AND MASS UNBAL (ALY=0.0001, MU2=0.00001)
0.4500    0.0001              500    15
    0.29                0.00                0.00
  0.00                70.8                  0.00
    0.00              0.00                 70.8
0.00001    0.0
 4800.0               0.0                 1600.0
  0.0                 16000.0             0.0
 1600.0               0.0                 15200.0
   0.00               0.00                 0.00
 0.0                   1000000.0           0.0
 0.0                  0.0                  1000000.0
    0.00               0.0                     0.00
0.0                    2000000.0           0.0
0.0                          0.0           2000000.0
  10.0                2.5                 2.5
 0.1                 0.1                  0.1
 0.00                0.00                  0.00
0.0                   600.0                0.0
0.0                  0.0                   600.0
  0.00                0.00                0.00
0.0                   800.0               0.0
0.0                     0.0                800.0
200.0      7.76       0.25    0.25        0.0001   0.00       0.0      0.0
-3.75      0.0        -2.0    0.1         0.0      450.0      450.0    450.0
0.05       0.065      0.125   0.1251      0.126    0.146      0.356    0.386
  0.386    0.0        9350.0  10500.0     0.0
1800.0     0.001
5.75                 0.0                   0.0
 0.0                   0.0                   32.175
 4.0                  0.0                  -2.0
-0.5                   0.0                 0.0
-6.583                 0.0                    0.0
0.0                    0.10                0.0
 0.25       0.25
2
0000000000000000000000
```

MENU DESCRIPTION

1 – TIME

2 – DELTA1 $\equiv \delta_1$, X_L-Displacement of point a in launcher basis $(X_{L_a} - X_{L_{a_o}})$.

3 – DELTA2 $\equiv \delta_2$, Y_L-Displacement of point a in launcher basis $(Y_{L_a} - Y_{L_{a_o}})$.

4 – DELTA $\equiv \delta_3$, Z_L-Displacement of point a in launcher basis $(Z_{L_a} - Z_{L_{a_o}})$.

5 – THETA(1) $\equiv \theta_1$, Rocket roll angle relative to launcher axis.

6 – THETA(2) $\equiv \theta_2$, Rocket pitch angle relative to launcher axes.

7 – THETA(3) $\equiv \theta_3$, Rocket yaw angle relative to launcher axes.

8 – ALPHA(1) $\equiv \alpha_1$, Launcher roll angle.

9 – ALPHA(2) $\equiv \alpha_2$, Launcher pitch angle.

10 – ALPHA(3) $\equiv \alpha_3$, Launcher yaw angle.

11 – DDELTA1 $\equiv \dot{\delta}_1$

12 – DDELTA2 $\equiv \dot{\delta}_2$

13 – DDELTA3 $\equiv \dot{\delta}_3$

14 – OMEGAXM $\equiv \Omega_1$, x_m-component of $\underline{\Omega}$ rocket angular velocity.

15 – OMEGAYM $\equiv \Omega_2$, y_m-component of $\underline{\Omega}$ rocket angular velocity.

16 – OMEGAZM $\equiv \Omega_3$, z_m-component of $\underline{\Omega}$ rocket angular velocity.

17 – OMEGAXL $\equiv \omega_1$, x_L-component of $\underline{\omega}$, launcher angular velocity.

18 – OMEGAYL $\equiv \omega_2$, y_L-component of $\underline{\omega}$, launcher angle velocity.

19 – OMEGAZL $\equiv \omega_3$, z_L-component of $\underline{\omega}$, launcher angular velocity.

20 – PHI $\equiv \Phi$, Rocket roll angle (absolute).

21 – THETA $\equiv \theta$, Rocket pitch angle (absolute).

22 – PSI $\equiv \psi$, Rocket yaw angle (absolute).

23 — x $\equiv$ X, X-coordinate of C_m.

24 — y $\equiv$ Y, Y-coordinate of C_m.

25 — z $\equiv$ Z, Z-coordinate of C_m.

26 — u $\equiv$ U, x_m-component of rocket's velocity.

27 — v $\equiv$ V, y_m-component of rocket's velocity.

28 — w $\equiv$ W, z_m-component of rocket's velocity.

29 — DPDELTA1 $\equiv \dot{\delta}_1$

30 — DPDELTA2 $\equiv \dot{\delta}_2$

31 — DPDELTA3 $\equiv \dot{\delta}_3$

32 — DOMEGAXY $\equiv \dot{\Omega}_1$

33 — DOMEGAYM $\equiv \dot{\Omega}_2$

34 — DOMEGAZM $\equiv \dot{\Omega}_3$

35 — DOMEGAXL $\equiv \dot{\omega}_1$

36 — DOMEGAYL $\equiv \dot{\omega}_2$

37 — DOMEGAZL $\equiv \dot{\omega}_3$

38 — DPHI $\equiv \dot{\phi}$

39 — DTHETA $\equiv \dot{\theta}$

40 — DPSI $\equiv \dot{\psi}$

41 — DX $\equiv \dot{X}$

42 — DY $\equiv \dot{Y}$

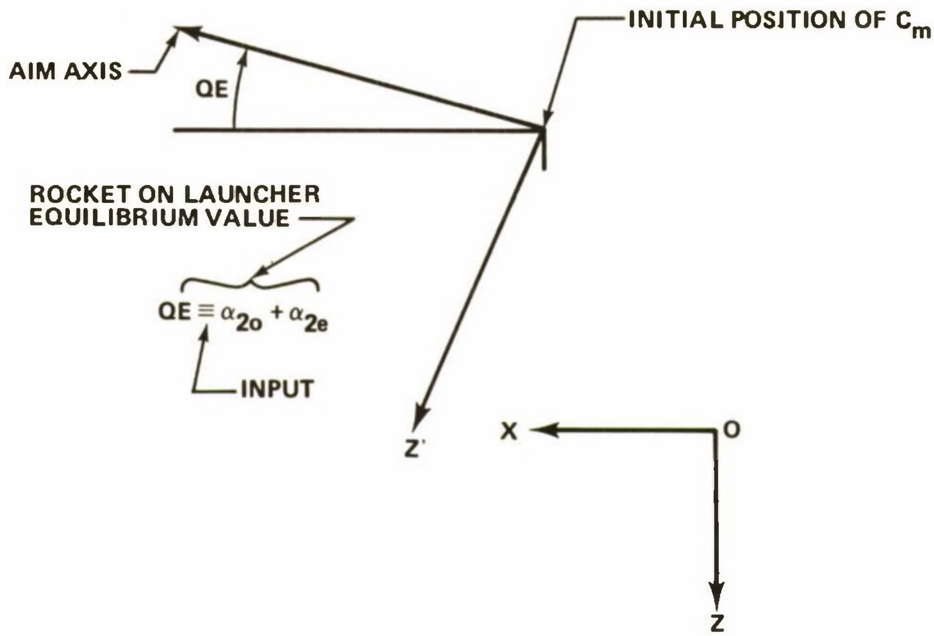
43 — DZ $\equiv \dot{Z}$

44 — DU $\equiv \dot{U}$

45 — DV $\equiv \dot{V}$

46 — DW $\equiv \dot{W}$

47 — DELTA Z $\equiv$ Displacement of C_m in the Z'-direction.



48 — DELDZ $\equiv \dot{Z} + U \sin(\alpha_{20} + \alpha_{2e})$ component velocity of C_m normal to the aim axis.

49 — DELTHETHA $\equiv \theta - \alpha_{2o} + \alpha_{2e} + \Delta\theta$

50 — SRSSA $\equiv \sqrt{\Delta\theta^2 \;\psi^2}$

SYMBOLS

$\underline{\underline{A}}$	Direction cosine matrix (launcher basis to rocket basis).
a	Point on rocket centerline at the aft shoes.
$\underline{\underline{B}}$	Direction cosine matrix (inertial basis to launcher basis).
$\underline{\underline{C}}_a, \underline{\underline{C}}_f, \underline{\underline{C}}_L$	Damping coefficient matrices.
$C_L x_L y_L z_L$	Centroidal launcher-fixed coordinate system.
$C_m x_m y_m z_m$	Centroidal rocket-fixed coordinate system.
$C_m x_p y_p z_p$	Principal, centroidal, rocket-fixed coordinate system.
$\underline{\underline{D}}$	Direction cosine matrix (centroidal rocket basis to principal, centroidal (rocket basis).
F	Magnitude of the thrust force.
$\underline{F}_{as}$	Force in the aft springs.
$\underline{F}_{Fa}$	Frictional force at aft shoes.
$\underline{F}_{Ff}$	Frictional force at forward shoes.
$\underline{F}_{fs}$	Force in the forward springs.
$\underline{F}_m$	Force on the rocket.
$\underline{F}_{m_D}$	Detent force on the rocket.
$\underline{F}_{m_F}$	Frictional force on the rocket.
F_{mg}	Gravitational force on the rocket.
$\underline{F}_{m_T}$	Thrust on the rocket (in launcher basis).
$\underline{F}_T$	Thrust (rocket basis).

f	Point on the centerline of the rocket at the forward shoes.
$\underline{g}$	Local acceleration of gravity.
$\hat{I},\hat{J},\hat{K}$	Inertial unit vectors.
$\underline{\underline{I}}_L$	Centroidal inertia tensor or matrix of the launcher.
$\underline{\underline{I}}_{L/0}$	Inertia tensor or matrix of the launcher about 0.
$\underline{\underline{I}}_m$	Centroidal inertia tensor or matrix of the rocket.
$\underline{\underline{I}}_{pm}$	Principal, centroidal inertia matrix of the rocket.
$\hat{i}_L,\hat{j}_L,\hat{k}_L$	Launcher-fixed unit vectors.
i_m, j_m, k_m	Rocket-fixed unit vectors.
$\underline{\underline{J}}$	Inertia matrix of the system about 0 during the spin up and detent phases.
$\underline{\underline{K}}_a, \underline{\underline{K}}_f, \underline{\underline{K}}_L$	Stiffness matrices.
$\underline{\ell}_F$	Vector from point a to a point on the line of action of the thrust, $\underline{F}_T$.
$\underline{\ell}_{fa}$	Vector along the centerline of the rocket directed from point a to point f.
M	Mass of the launcher.
m	Mass of the rocket.
OXYZ	Inertial coordinate system with origin at the launcher pivot point.
$\underline{R}$	Vector from 0 to C_L.
$\underline{S}_a = x_{L_a}\hat{i}_L + y_{L_a}\hat{j}_L + z_{L_a}\hat{k}_L$	Vector from C_L to a.
$\underline{T}_{m/a}$	Torque on the rocket about a.

$\underline{T}_{m/C_m}$	Torque on the rocket about C_m.
$\underline{T}_o$	External torque on the system about 0.
$\underline{T}_s$	Spin torque.
$\underline{T}_{f/C_m}$	Frictional torque on the rocket about C_m.
t	Time.
U,V,W	Rocket-fixed components of the rocket's velocity.
$\underline{u} = u_1\hat{i}_m + \epsilon_y\hat{j}_m + \epsilon_z\hat{k}_m$	Vector from a to C_m.
$\underline{V}$	Velocity of the rocket.
X,Y,Z	Inertial coordinates of C_m.

<u>Greek Characters</u>

$\alpha_1,\alpha_2,\alpha_3$	Eulerian angles used in defining the orientation of the $C_Lx_Ly_Lz_L$ system.
$\underline{\alpha} = (\alpha_1\ \alpha_2\ \alpha_3)^T$	
α_y,α_z	Thrust malalignment angles.
$\delta_1,\delta_2,\delta_3$	Displacements of point a from its undeformed position.
ϵ_y,ϵ_z	Displacements of C_m from the rocket centerline.
$\zeta_{L_X},\zeta_{L_Y},\zeta_{L_Z}$	Launcher damping ratios.
θ	Rocket pitch angle.
$\theta_1,\theta_2,\theta_3$	Eulerian angles used in defining the orientation of the $C_mx_my_mz_m$ system relative to the $C_Lx_Ly_Lz_L$ system.
μ	Friction coefficient (sliding).
μ_s	Spin bearing friction coefficient.

σ_y, σ_z	Linear thrust malalignment parameters.
ϕ	Missile roll angle.
ψ	Missile yaw angle.
$\underline{\Omega} = \Omega_1 \hat{i}_m + \Omega_2 \hat{j}_m + \Omega_3 \hat{k}_m$	Rocket angular velocity.
$\underline{\omega} = \omega_1 \hat{i}_L + \omega_2 \hat{j}_L + \omega_3 \hat{k}_L$	Launcher angular velocity.
$\omega_{n_{L_x}}, \omega_{n_{L_y}}, \omega_{n_{L_z}}$	Natural frequencies of the launcher.
$(\ )_L$	Launcher.
$(\ )_m$	Rocket.
$(\ )_o$	Initial value or undeformed value.
$(\ )_e$	Change needed to achieve equilibrium.
$(\ )^{-1}$	Inverse.
$(\ )^T$	Transpose.
ρ_f, ρ_2	Distances from the rocket centerline at points f and a, respectively, to the rocket/launcher interface (rail, tube wall, etc.)

Other Notations

$\sim$ Used above a vector (3 × 1 matrix), say $\underline{\Omega}$, to denote the skew-symmetric matrix,

$$\tilde{\underline{\underline{\Omega}}} = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} .$$

o Used above a vector to indicate that the time derivatives of only the components of the vector are to be taken.

$\cdot$	Used above a vector or scalar to denote total time differentation.
$\hat{}$	Denotes a unit vector.
$\_$	Denotes a vector or column matrix.
$=$	Denotes a tensor or square matrix.
$\underline{\underline{E}}$	Identity matrix (3×3).

$$\underline{\underline{E}}_1 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

$$\underline{\underline{E}}_{23} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

$\overline{L},\overline{m}$ Used with a vector with a circle above it to denote the system in which the component derivative is taken; e.g.,

$$\underset{m}{\overset{\circ}{\underline{\Omega}}} = \dot{\Omega}_1 \hat{i}_m + \dot{\Omega}_2 \hat{j}_m + \dot{\Omega}_3 \hat{k}_m \text{ and } \underset{L}{\overset{\circ}{\underline{\omega}}} = \dot{\omega}_1 \hat{i}_L + \dot{\omega}_2 \hat{j}_L + \dot{\omega}_3 \hat{k}_L m \quad .$$

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